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Trends in Mathematical Sciences

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Mathematical Modelling and Simulations of Successive Waves of COVID-19 in Brazil

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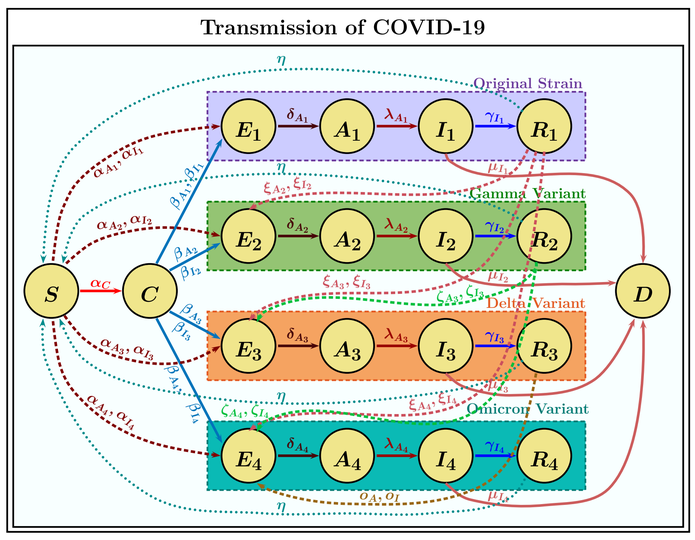
INTRODUCTION

Since December 2019, the world has been facing one of the most devastating pandemics in history, the COVID-19, caused by the Sars-CoV-2 virus, whose main form of transmission is between people, through droplets of saliva expelled by an infected person by coughing or sneezing. Thus, the WHO's recommendations were the adoption of hygiene and respiratory safety measures, such as the use of masks, social distancing, reducing the movement of people and, in some places, the lockdown strategy. Such strategies were effective in many countries, but in others, such as Brazil, they were not implemented efficiently, which resulted in inadequate management of the pandemic, resulting in successive waves of the disease. In this work, we present the foundations of the theory of epidemiological models, as well as some case studies, and based on the SIR model, we build other more elaborate models, according to the evolution of the COVID-19 pandemic situation, in order to include the Gamma, Delta and Omicron variants, using the Heaviside step function, and we also analysed the vaccination situations.
For all models, we use a modification of the schemes and parameters proposed by [1] and [2] and the data from [3] and [4], we performed computer simulations and compared the curve obtained with real data from the active cases, which reinforced the model's efficiency in describing the behaviour of the pandemic and highlighted the importance of population vaccination in controlling and reducing cases.

MATHEMATICAL MODEL

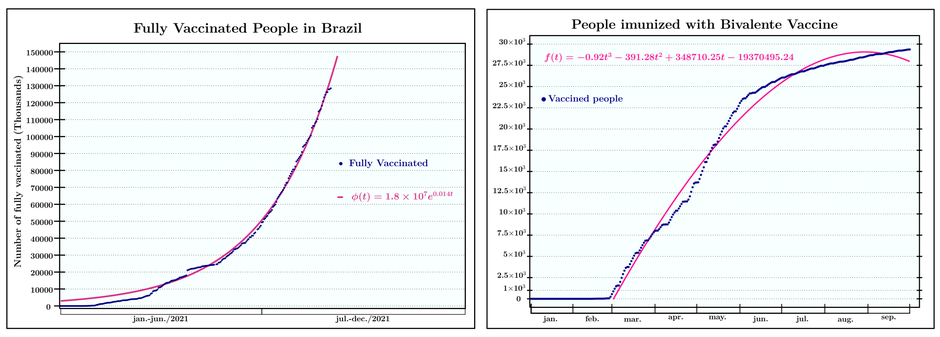


Relationships between the classes of individuals for the study of the dynamics of COVID-19.

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \eta[R_1(t) + R_2(t) + R_3(t) + R_4(t)] - S(t)[\alpha_C + \alpha_{A_1}A_1(t) + \alpha_{I_1}I_1(t) + \alpha_{A_2}A_2(t) + \alpha_{I_2}I_2(t) + \alpha_{A_3}A_3(t) + \alpha_{I_3}I_3(t) + \alpha_{A_4}A_4(t) + \alpha_{I_4}I_4(t)]$$
$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = \alpha_C S(t) - C(t)[\beta_{A_1}A_1(t) + \beta_{I_1}I_1(t) + \beta_{A_2}A_2(t) + \beta_{I_2}I_2(t) + \beta_{A_3}A_3(t) + \beta_{I_3}I_3(t) + \beta_{A_4}A_4(t) + \beta_{I_4}I_4(t)]$$
$$\frac{\mathrm{d}E_1(t)}{\mathrm{d}t} = \alpha_{A_1}S(t)A_1(t) + \alpha_{I_1}S(t)I_1(t) + \beta_{A_1}C(t)A_1(t) + \beta_{I_1}C(t)I_1(t - \delta_{A_1}E_1(t)$$
$$\frac{\mathrm{d}A_1(t)}{\mathrm{d}t} = \delta_{A_1}E_1(t) - \lambda_{A_1}A_1(t)$$
$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = \lambda_{A_1}A_1(t) - \gamma_{I_1}I_1(t) - \mu_{I_1}I_1(t)$$
$$\frac{\mathrm{d}R_1(t)}{\mathrm{d}t} = \gamma_{I_1}I_1(t) - \eta R_1(t) - [\xi_{A_2}A_2(t) + \xi_{I_2}I_2(t) + \xi_{A_3}A_3(t) + \xi_{I_3}I_3(t) + \xi_{A_4}A_4(t) + \xi_{I_4}I_4(t)]R_1(t)$$
$$\frac{\mathrm{d}E_2(t)}{\mathrm{d}t} = H(t-\tau_2)\{S(t)[\alpha_{A_2}A_2(t) + \alpha_{I_2}I_2(t)] + C(t)[\beta_{A_2}A_2(t) + \beta_{I_2}I_2(t)] + R_1(t)[\xi_{A_2}A_2(t) + \xi_{I_2}I_2(t)] - \delta_{A_2}E_2(t)\}$$
$$\frac{\mathrm{d}A_2(t)}{\mathrm{d}t} = H(t-\tau_2)[\delta_{A_2}E_2(t) - \lambda_{A_2}A_2(t)]$$
$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = H(t-\tau_2)[\lambda_{A_2}A_2(t) - \gamma_{I_2}I_2(t) - \mu_{I_2}I_2(t)]$$
$$\frac{\mathrm{d}R_2(t)}{\mathrm{d}t} = H(t-\tau_2)\{\gamma_{I_2}I_2(t) - \eta R_2(t) - [\zeta_{A_3}A_3 + \zeta_{I_3}I_3 + \zeta_{A_4}A_4 + \zeta_{I_4}I_4]R_2(t)\}$$
$$\frac{\mathrm{d}E_3(t)}{\mathrm{d}t} = H(t-\tau_3)\{S(t)[\alpha_{A_3}A_3(t) + \alpha_{I_3}I_3(t)] + C(t)[\beta_{A_3}A_3(t) + \beta_{I_3}I_3(t)] + [\xi_{A_3}A_3(t) + \xi_{I_3}I_3(t)]R_1(t) + [\zeta_{A_3}A_3 + \zeta_{I_3}I_3]R_2(t) - \delta_{A_3}E_3(t)\}$$
$$\frac{\mathrm{d}A_3(t)}{\mathrm{d}t} = H(t-\tau_3)[\delta_{A_3}E_3(t) - \lambda_{A_3}A_3(t)]$$
$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = H(t-\tau_3)[\lambda_{A_3}A_3(t) - \gamma_{I_3}I_3(t) - \mu_{I_3}I_3(t)]$$
$$\frac{\mathrm{d}R_3(t)}{\mathrm{d}t} = H(t-\tau_3)\{\gamma_{I_3}I_3(t) - [\eta + o_A + o_I]R_3(t)\}$$
$$\frac{\mathrm{d}E_4(t)}{\mathrm{d}t} = H(t-\tau_4)\{S(t)[\alpha_{A_4}A_4(t) + \alpha_{I_4}I_4(t)] + C(t)[\beta_{A_4}A_4(t) + \beta_{I_4}I_4(t)] + [\xi_{A_4}A_4(t) + \xi_{I_4}I_4(t)]R_1(t) + [\zeta_{A_4}A_4 + \zeta_{I_4}I_4]R_2(t) + [o_A A_4 + o_I I_4]R_3(t) - \delta_{A_4}E_4(t)\}$$
$$\frac{\mathrm{d}A_4(t)}{\mathrm{d}t} = H(t-\tau_4)[\delta_{A_4}E_4(t) - \lambda_{A_4}A_4(t)]$$
$$\frac{\mathrm{d}I_4(t)}{\mathrm{d}t} = H(t-\tau_4)[\lambda_{A_4}A_4(t) - \gamma_{I_4}I_4(t) - \mu_{I_4}I_4(t)]$$
$$\frac{\mathrm{d}R_4(t)}{\mathrm{d}t} = H(t-\tau_4)[\gamma_{I_4}I_4(t) - \eta R_4(t)]$$
$$\frac{\mathrm{d}D(t)}{\mathrm{d}t} = \mu_{I_1}I_1(t) + H(t-\tau_2)\mu_{I_2}I_2(t) + H(t-\tau_3)\mu_{I_3}I_3(t) + H(t-\tau_4)\mu_{I_4}I_4(t).$$

Vaccination

- Original strain, Gama and Delta variants: CoronaVac®, AZD1222 (ChAdOx1-S nCoV-19)®, Comirnaty® and Janssen Ad26.COV2.S®
- Omicron variant: Comirnaty® Bivalente



Number of vaccinated people with de normal vaccine and Bivalente.

Reduction of the number of infected people:

- Original strain, Gama and Delta variants: started in January of 2021 (day 400) and with mean average efficacy rate of $e = 75.3\%$

$$\psi(t) = H(t - 400)e\frac{\phi(t)}{N},$$

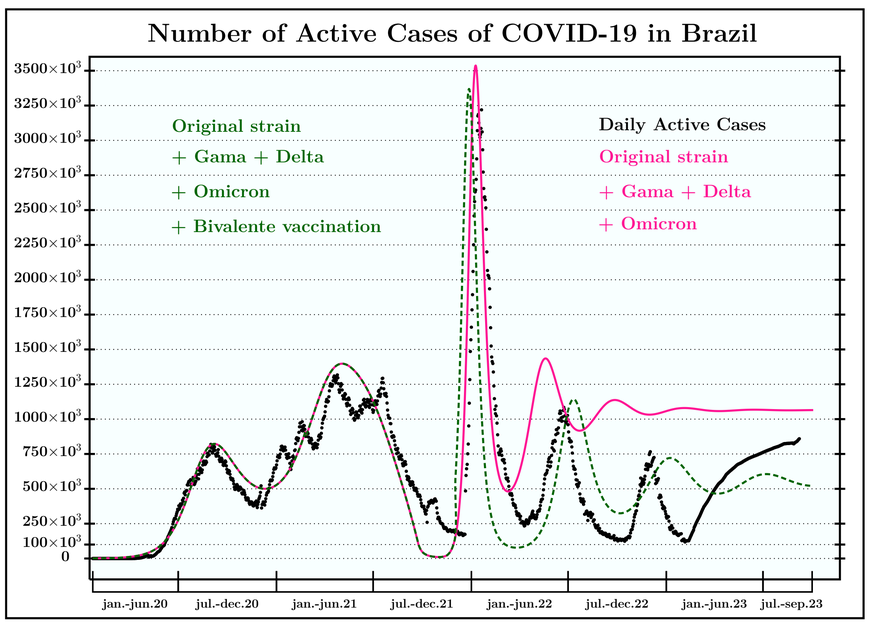
where $H(t)$ is the Heaviside step function, $\phi(t)$ is the function that describes the complete immunization curve and N is the total population of Brazil.

- Omicron variant: started in January of 2022 (day 730) and with efficacy rate of 95%

$$\phi(t) = H(t - 730)e\frac{f(t)}{N},$$

where $H(t)$ is the Heaviside step function, $f(t)$ is the function that describes the complete immunization curve and N is the total population of Brazil.

RESULTS



Active cases of COVID-19 and the simulation the Gamma, Delta and Omicron Variants, including vaccination.

ACKNOWLEDGMENTS

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FINAL REMARKS

Using behavioural schemes and systems of differential equations, we present efficient epidemiological models in describing the COVID-19 pandemic in Brazil, as confirmed by the graphs obtained through numerical simulations. The technique proposed for the inclusion of new variants, through the addition of compartments, can be easily used to simulate new scenarios if new variants appear and also if there is a change in the form of immunization of the population. However, the proposed models do not consider the vital dynamics, since the simulations for a period of four years, and do not consider the population's behavioural changes, which results in a small difference between the curves obtained and the real data.

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