Eigenvalue problems on graphs and hypergraphs

Machine Learning and PDEs Workshop

Daniel Tenbrinck
FAU Erlangen-Nürnberg
April 28th, 2025

Outline

Weighted graphs in data science and machine learning

- Data modeling with graphs
- Differential operators on graphs

Eigenvalue problems on graphs

- Linear eigenvalue problems
- Nonlinear eigenvalue problems

Extension to hypergraphs

- Data modeling with hypergraphs
- Differential operators on hypergraphs
- Eigenvalue problems on hypergraphs

Outline

Weighted graphs in data science and machine learning

- Data modeling with graphs
- Differential operators on graphs

Eigenvalue problems on graphs

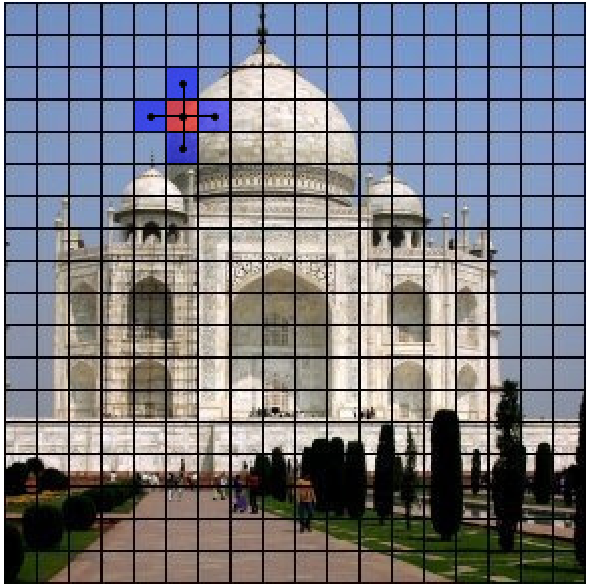
- Linear eigenvalue problems
- Nonlinear eigenvalue problems

Extension to hypergraphs

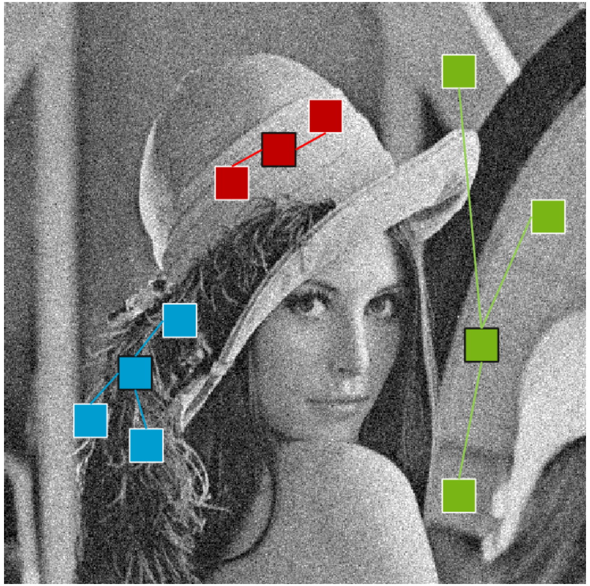
- Data modeling with hypergraphs
- Differential operators on hypergraphs
- Eigenvalue problems on hypergraphs

Discrete data modeling using weighted graphs

Question: How can we apply graphs for image processing ?



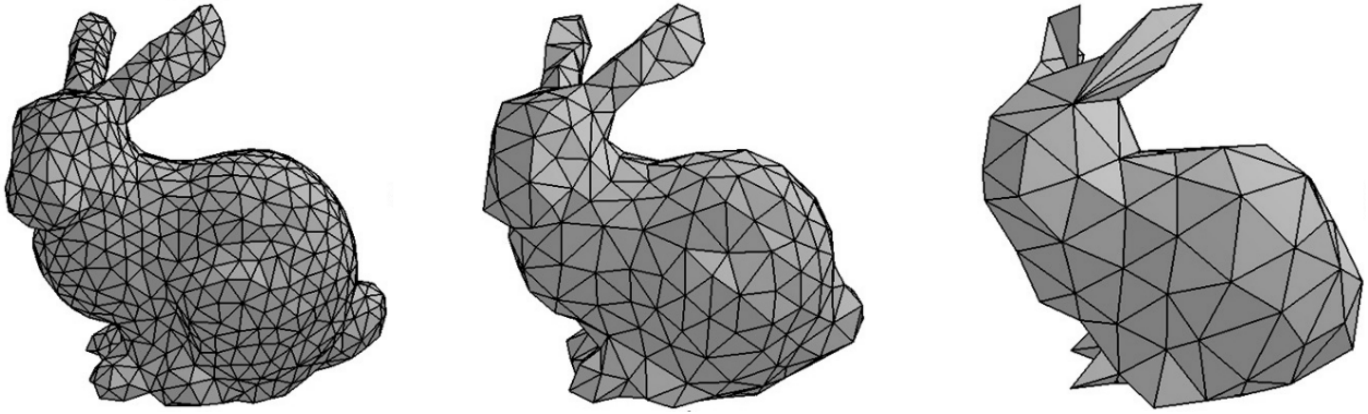
Local neighborhood of a pixel



Nonlocal neighborhood of a pixel

Discrete data modeling using weighted graphs

Question: How can we apply graphs for polygon mesh processing ?

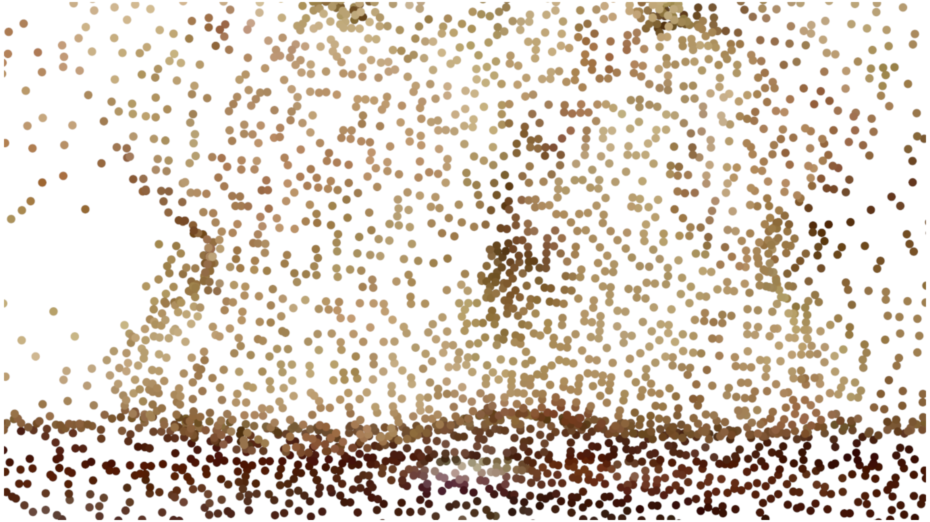


Polygon mesh approximation of a 3D surface.

[1] Image: Gabriel Peyré

Discrete data modeling using weighted graphs

Question: How can we apply graphs for point cloud processing?

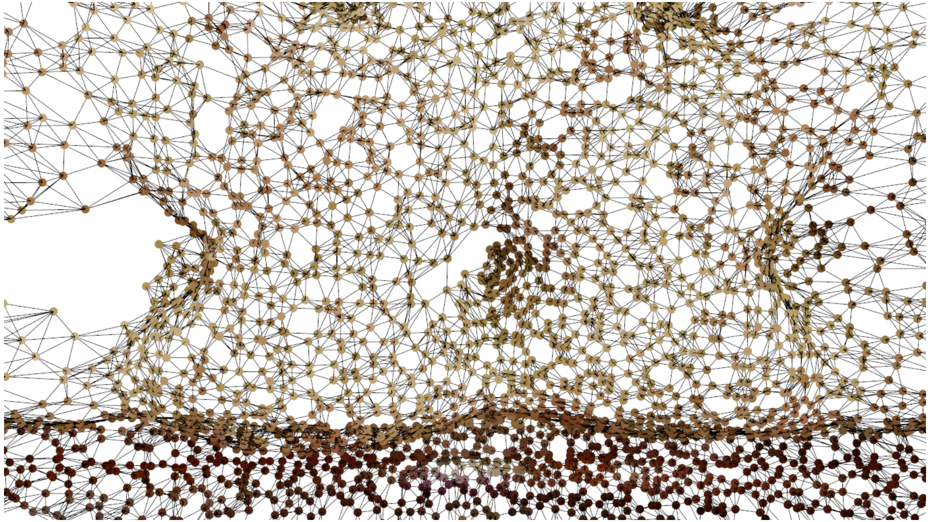


Colored 3D point cloud data of a scanned chair

[1] Image: Abderrahim Elmoataz

Discrete data modeling using weighted graphs

Question: How can we apply graphs for point cloud processing?

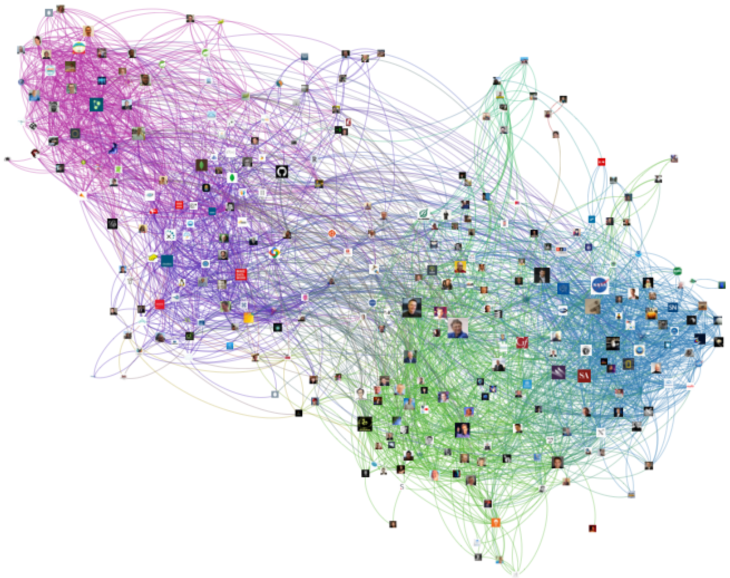


Graph construction on a 3D point cloud

[1] Image: Abderrahim Elmoataz

Discrete data modeling using weighted graphs

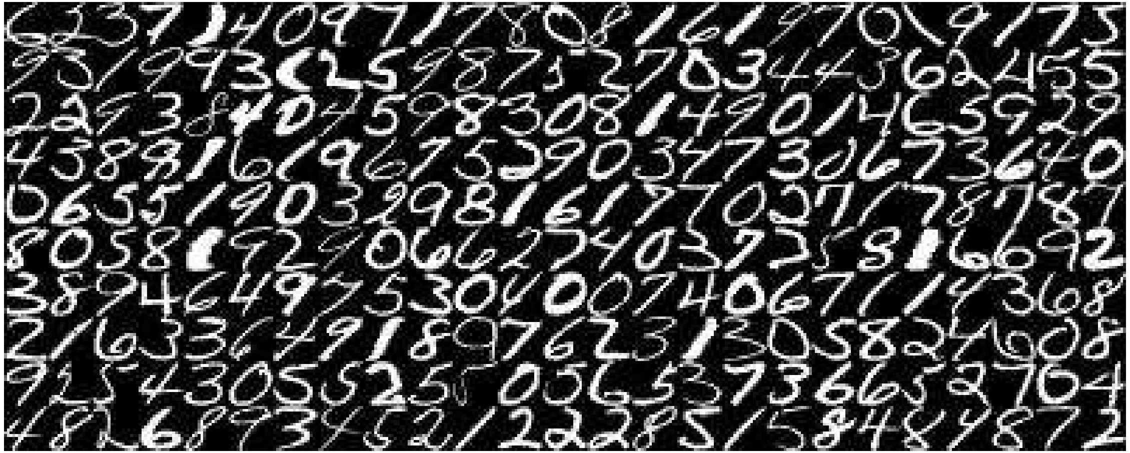
Question: How can we apply graphs for social network analysis?



[1] Image: Caleb Jones

Discrete data modeling using weighted graphs

Question: How can we apply graphs for machine learning?

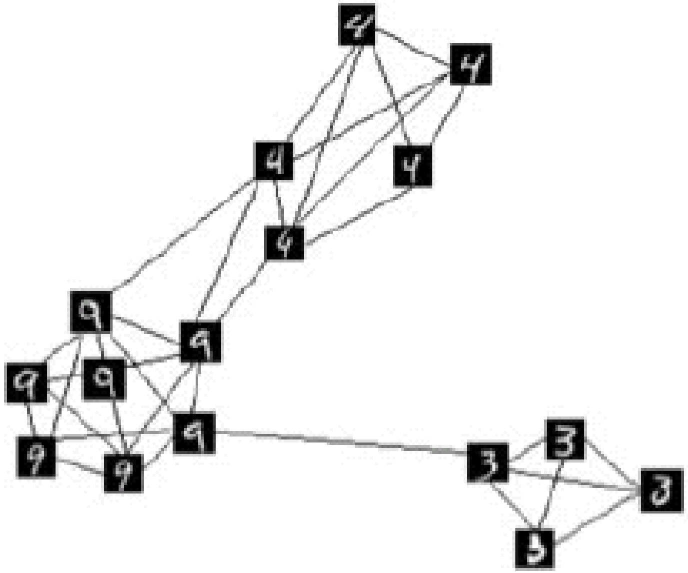


Subset of handwritten digits from MNIST database[2] Graph construction using suitable similarity features

[2] Y. LeCun, C. Cortes, C. Burges: *The MNIST database of handwritten digits*

Discrete data modeling using weighted graphs

Question: How can we apply graphs for machine learning?



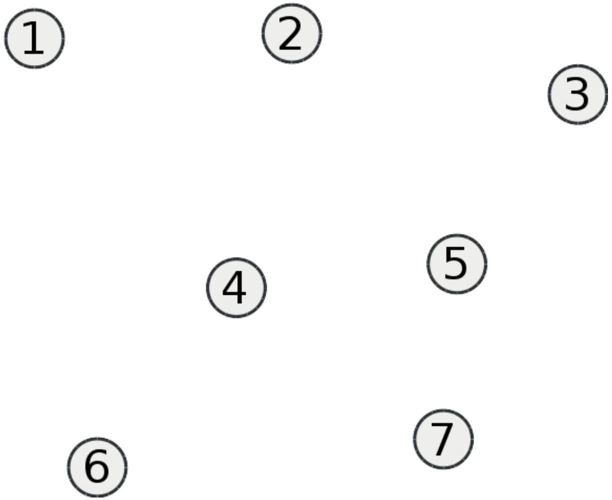
Finite weighted graphs

An undirected finite weighted graph $G = (V, E, w)$ consists of:

Finite weighted graphs

An undirected finite weighted graph $G = (V, E, w)$ consists of:

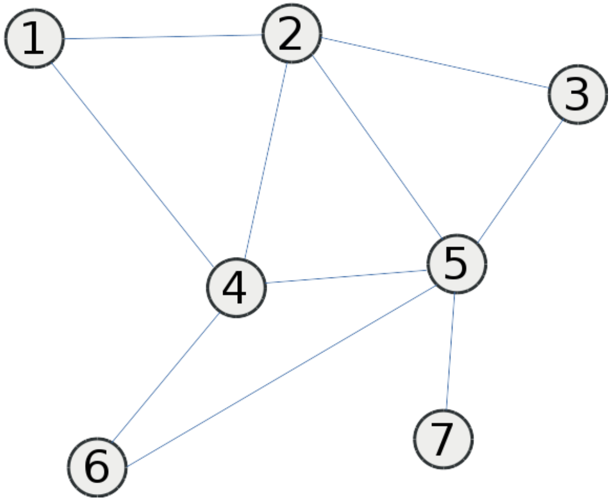
- a finite set of vertices $V = (v_1, \dots, v_n)$



Finite weighted graphs

An undirected finite weighted graph $G = (V, E, w)$ consists of:

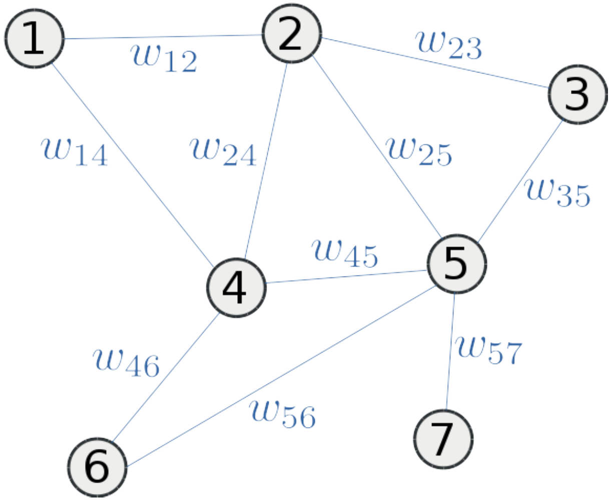
- a finite set of vertices $V = (v_1, \ldots, v_n)$
- a finite set of edges $E \subset V \times V$, $\quad (u, v) \in E \rightarrow$ short: $v \sim u$



Finite weighted graphs

An undirected finite weighted graph $G = (V, E, w)$ consists of:

- a finite set of vertices $V = (v_1, \ldots, v_n)$
- a finite set of edges $E \subset V \times V$, $\quad (u, v) \in E \to$ short: $v \sim u$
- a weight function $w \colon E \to [0, 1]$ with: $w(u, v) > 0 \Leftrightarrow (u, v) \in E$



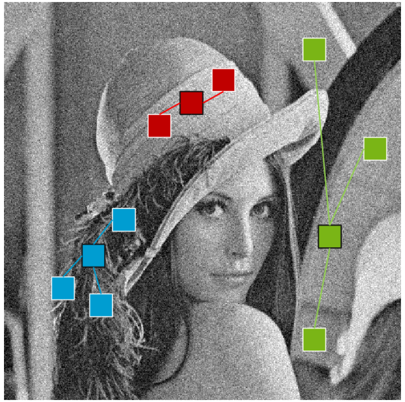
Finite weighted graphs

Application data can be represented by a vertex function $f\colon V \to \mathbb{R}^m$, e.g.,

Finite weighted graphs

Application data can be represented by a vertex function $f\colon V \to \mathbb{R}^m$, e.g.,

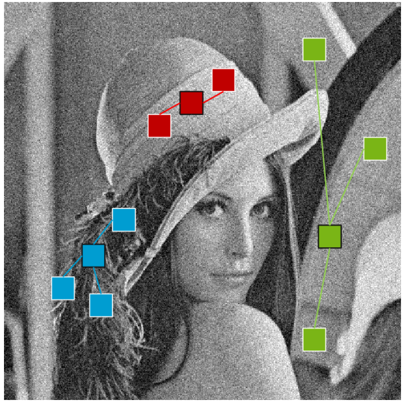
- Grayscale values or RGB colors

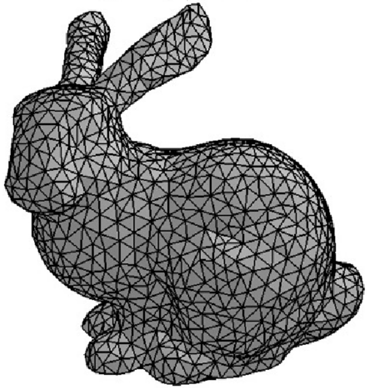


Finite weighted graphs

Application data can be represented by a vertex function $f\colon V \to \mathbb{R}^m$, e.g.,

- Grayscale values or RGB colors
- 3D coordinates

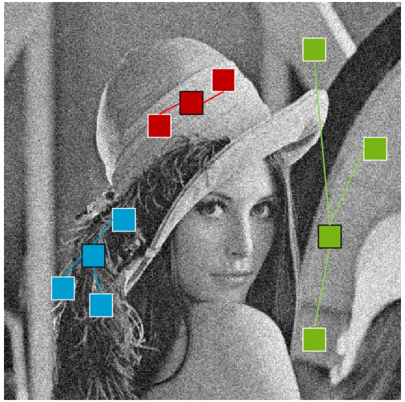
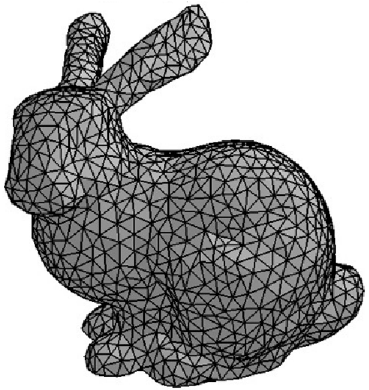
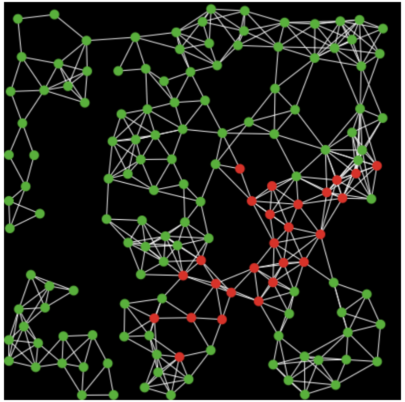




Finite weighted graphs

Application data can be represented by a vertex function $f\colon V \to \mathbb{R}^m$, e.g.,

- Grayscale values or RGB colors
- 3D coordinates
- Labels
- …

Weighted gradient and divergence operator

Let (V, E, w) be a graph, $f \in H(V)$ a vertex function and $G \in H(E)$ an edge function.

Weighted gradient and divergence operator

Let (V, E, w) be a graph, $f \in H(V)$ a vertex function and $G \in H(E)$ an edge function.

The gradient $\nabla \colon H(V) \to H(E)$ of $f \in H(V)$ along an edge $(u, v) \in E$ is given as:

$$\nabla f(u, v) \; = \; \sqrt{w(u,v)}(f(v) - f(u)) \tag{1}$$

[1] A. Elmoataz, O. Lézoray, S. Bougleux: *Nonlocal Discrete Regularization on Weighted Graphs*. IEEE TIP 17 (2008)
G. Gilboa, S. Osher: *Nonlocal Operators with Applications to Image Processing*. Multiscale Model. Simul. 7 (2008)

Weighted gradient and divergence operator

Let (V, E, w) be a graph, $f \in H(V)$ a vertex function and $G \in H(E)$ an edge function.

The gradient $\nabla\colon H(V) \to H(E)$ of $f \in H(V)$ along an edge $(u, v) \in E$ is given as:

$$\nabla f(u, v) \; = \; \sqrt{w(u, v)}(f(v) - f(u)) \tag{1}$$

One can deduce the adjoint operator $\nabla^*\colon H(E) \to H(V)$ of ∇ by the property:

$$\langle \nabla f, G \rangle_{H(E)} \; = \; \langle f, \nabla^* G \rangle_{H(V)} \tag{2}$$

[1] A. Elmoataz, O. Lézoray, S. Bougleux: *Nonlocal Discrete Regularization on Weighted Graphs*. IEEE TIP 17 (2008)
G. Gilboa, S. Osher: *Nonlocal Operators with Applications to Image Processing*. Multiscale Model. Simul. 7 (2008)

Weighted gradient and divergence operator

Let (V, E, w) be a graph, $f \in H(V)$ a vertex function and $G \in H(E)$ an edge function.

The gradient $\nabla \colon H(V) \to H(E)$ of $f \in H(V)$ along an edge $(u, v) \in E$ is given as:

$$\nabla f(u, v) = \sqrt{w(u, v)}(f(v) - f(u)) \tag{1}$$

One can deduce the adjoint operator $\nabla^* \colon H(E) \to H(V)$ of ∇ by the property:

$$\langle \nabla f, G \rangle_{H(E)} = \langle f, \nabla^* G \rangle_{H(V)} \tag{2}$$

Then the divergence $\operatorname{div} \colon H(E) \to H(V)$ of G in a vertex $u \in V$ is given as:

$$\operatorname{div} G(u) = -\nabla^* G(u) = \sum_{v \sim u} \sqrt{w(u, v)}(G(u, v) - G(v, u)) \tag{3}$$

[1] A. Elmoataz, O. Lézoray, S. Bougleux: *Nonlocal Discrete Regularization on Weighted Graphs.* IEEE TIP 17 (2008)
G. Gilboa, S. Osher: *Nonlocal Operators with Applications to Image Processing.* Multiscale Model. Simul. 7 (2008)

Translating variational problems to graphs

Example: A general variational data denoising model

Translating variational problems to graphs

Example: A general variational data denoising model

Find a minimizer $f : V \to \mathbb{R}^m$ of the energy functional

$$E(f) \;=\; \lambda||f - f_0||^2 + ||\nabla f||_{p,q}^p \,, \qquad \lambda > 0$$

Translating variational problems to graphs

Example: A general variational data denoising model

Find a minimizer $f : V \to \mathbb{R}^m$ of the energy functional

$$E(f) \ = \ \lambda||f - f_0||^2 + ||\nabla f||_{p,q}^p \, , \quad \lambda > 0$$

with $||\nabla f||_{p,q} \ = \ \left(\frac{1}{p} \sum_{u \in V} \Big(\sum_{v \sim u} ||\nabla f(u,v)||^q \Big)^{\frac{p}{q}} \right)^{\frac{1}{p}} , \quad 1 \leq p, q < \infty.$

Translating variational problems to graphs

Example: A general variational data denoising model

Find a minimizer $f : V \to \mathbb{R}^m$ of the energy functional

$$E(f) \;=\; \lambda||f - f_0||^2 + ||\nabla f||_{p,q}^p \,, \qquad \lambda > 0$$

with $$||\nabla f||_{p,q} \;=\; \left(\frac{1}{p} \sum_{u \in V} \Big(\sum_{v \sim u} ||\nabla f(u,v)||^q \Big)^{\frac{p}{q}} \right)^{\frac{1}{p}}, \quad 1 \leq p,q < \infty.$$

Special cases:

p=q=1 : (Anisotropic) total variation regularization

p=q=2 : Generalized Tikhonov regularization

Graph p-Laplace operators

Observation:

Optimality conditions for the p-Dirichlet energy lead to the graph p-Laplace operator:

$$\Delta_p f(u) \;=\; \operatorname{div}\left(|\nabla f|^{p-2}\nabla f\right)(u) \;=\; \sum_{v \sim u} (w(u,v))^{p/2} |f(v) - f(u)|^{p-2} (f(v) - f(u))$$

[2] A. Elmoataz, M. Toutain, D. Tenbrinck: *On the p-Laplacian and ∞-Laplacian on Graphs with Applications in Image and Data Processing*. SIAM JIS 8 (2016)

Graph p-Laplace operators

Observation:

Optimality conditions for the p-Dirichlet energy lead to the graph p-Laplace operator:

$$\Delta_p f(u) \; = \; \operatorname{div}\left(|\nabla f|^{p-2} \nabla f\right)(u) \; = \; \sum_{v \sim u} (w(u,v))^{p/2} |f(v) - f(u)|^{p-2} (f(v) - f(u))$$

Special case for $p = 2$:

$$\Delta f(u) \; = \; \operatorname{div}(\nabla f)(u) \; = \; \sum_{v \sim u} w(u,v)(f(v) - f(u))$$

[2] A. Elmoataz, M. Toutain, D. Tenbrinck: *On the p-Laplacian and ∞-Laplacian on Graphs with Applications in Image and Data Processing.* SIAM JIS 8 (2016)

Graph p-Laplace operators

Observation:

Optimality conditions for the p-Dirichlet energy lead to the graph p-Laplace operator:

$$\Delta_p f(u) \;=\; \operatorname{div}\left(|\nabla f|^{p-2}\nabla f\right)(u) \;=\; \sum_{v\sim u}(w(u,v))^{p/2}|f(v)-f(u)|^{p-2}(f(v)-f(u))$$

Special case for $p = 2$:

$$\Delta f(u) \;=\; \operatorname{div}(\nabla f)(u) \;=\; \sum_{v\sim u} w(u,v)(f(v)-f(u))$$

The graph Laplace operator is linear and can be expressed as matrix $L \in \mathbb{R}^{n\times n}$ via

$$L \;=\; D - W, \tag{4}$$

where D is the **degree matrix** and W is the **weight matrix** of the weighted graph.

[2] A. Elmoataz, M. Toutain, D. Tenbrinck: *On the p-Laplacian and ∞-Laplacian on Graphs with Applications in Image and Data Processing*. SIAM JIS 8 (2016)

Outline

Weighted graphs in data science and machine learning

- Data modeling with graphs
- Differential operators on graphs

Eigenvalue problems on graphs

- Linear eigenvalue problems
- Nonlinear eigenvalue problems

Extension to hypergraphs

- Data modeling with hypergraphs
- Differential operators on hypergraphs
- Eigenvalue problems on hypergraphs

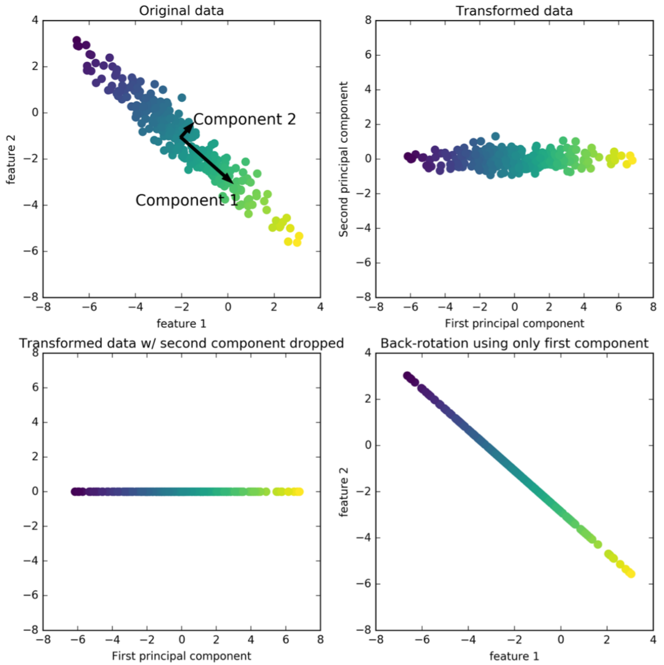
Principal component analysis

Aim:
Dimension reduction / data simplification

Eigenvalue problem:
Compute the biggest eigenvalues and corresponding eigenvectors via

$$\Sigma x = \lambda x,$$

for which $\Sigma = \mathbb{E}[XX^T]$ is the covariance matrix of the given (centralized) data X.



[3] Image courtesy: Ashwin N

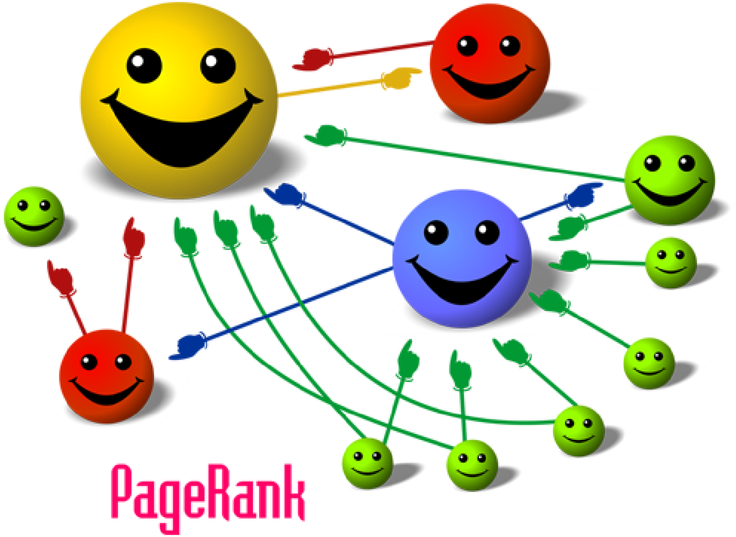
PageRank algorithm[5]

Aim:
Recommendation system for websites based on HTML links

Eigenvalue problem:
Compute the biggest eigenvalue and corresponding eigenvector via

$$Gx = \lambda x,$$

for which $G = \beta M + \frac{(1-\beta)}{n}\mathbb{1}$ is the Google matrix.



[4]Image courtesy: Wikmedia – Felipe Micaroni Lalli
[5]L. Page, S. Brin, R. Motwani, T. Winograd: *The PageRank Citation Ranking: Bringing Order to the Web.* Technical Report (1999)

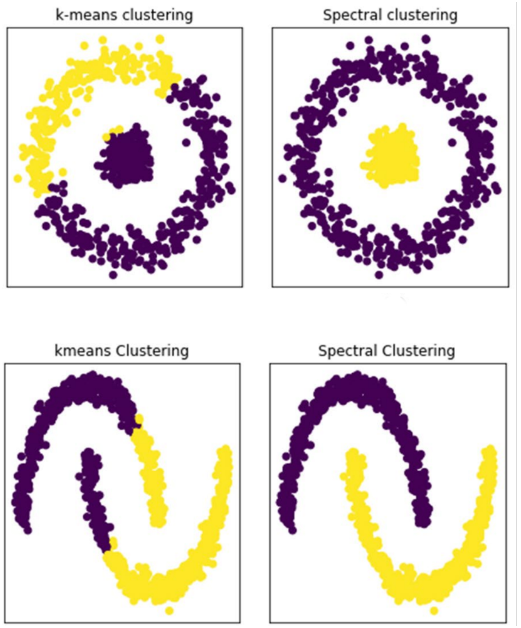
Spectral clustering[7]

Aim:
Unsupervised data clustering

Eigenvalue problem:
Compute the second smallest eigenvalue and corresponding eigenvector via

$$Lx = \lambda x,$$

for which $L = D - W$ is the graph Laplacian matrix of the given graph.



[6]Image: M. El Hamri, Y. Bennani, I. Falih: *Hierarchical optimal transport for unsupervised domain adaptation*. Machine Learning 111:4159–4182 (2022)
[7]U. von Luxburg: *A Tutorial on Spectral Clustering*. arXiv:0711.0189 (2007)

What is a nonlinear eigenvalue problem?

What is a nonlinear eigenvalue problem?

Aim:

Find an element u of a suitable space X and a scalar $\lambda \in \mathbb{R}$ s.t. for two (possibly) nonlinear functionals $T, S \colon X \to \mathbb{R}$ the following equation is fulfilled:

$$T(u) \; = \; \lambda S(u) \tag{5}$$

What is a nonlinear eigenvalue problem?

Aim:

Find an element u of a suitable space X and a scalar $\lambda \in \mathbb{R}$ s.t. for two (possibly) nonlinear functionals $T, S \colon X \to \mathbb{R}$ the following equation is fulfilled:

$$T(u) = \lambda S(u) \tag{5}$$

Interesting case:[8]

- X is reflexive Banach space
- $T(u) \coloneqq \partial J(u) \in X^*$ with J as proper, l.s.c., convex functional
- $S(u) \coloneqq \partial H(u) \in X^*$ with H as proper, l.s.c., convex, abs. p-homogeneous functional

[8] L. Bungert, M. Burger: *Gradient Flows and Nonlinear Power Methods for the Computation of Nonlinear Eigenfunctions*, Handbook of Numerical Analysis (2021)

What is a nonlinear eigenvalue problem?

Aim:
Find an element u of a suitable space X and a scalar $\lambda \in \mathbb{R}$ s.t. for two (possibly) nonlinear functionals $T, S \colon X \to \mathbb{R}$ the following equation is fulfilled:

$$T(u) \;=\; \lambda S(u) \tag{5}$$

Interesting case:[8]

- X is reflexive Banach space
- $T(u) \coloneqq \partial J(u) \in X^*$ with J as proper, l.s.c., convex functional
- $S(u) \coloneqq \partial H(u) \in X^*$ with H as proper, l.s.c., convex, abs. p-homogeneous functional

Example:
For $p > 1$, $X \coloneqq W_0^{1,p}(\Omega)$, $J(u) \coloneqq \frac{1}{p} \int_\Omega ||\nabla u(x)||^p \mathrm{d}x$ and $H(u) \coloneqq \frac{1}{p}||u||^p_{L^p(\Omega)}$ we get:

$$-\Delta_p u(x) \;=\; \lambda |u(x)|^{p-2} u(x) \tag{6}$$

[8] L. Bungert, M. Burger: *Gradient Flows and Nonlinear Power Methods for the Computation of Nonlinear Eigenfunctions*, Handbook of Numerical Analysis (2021)

How to compute nonlinear eigenfunctions?

Flow-based approaches:

$$u'(t) = -\nabla J(u(t))$$

- R. Hynd, E. Lindgren: *Approximation of the least Rayleigh quotient for degree p homogeneous functionals.* Journal of Functional Analysis 272 (2017)
- G. Gilboa: *Iterative Methods for Computing Eigenvectors of Nonlinear Operators*. Handbook of Mathematical Models and Algorithms in Computer Vision and Imaging (2021)
- L. Bungert, M. Burger, D. Tenbrinck: *Computing Nonlinear Eigenfunctions via Gradient Flow Extinction*

How to compute nonlinear eigenfunctions?

Flow-based approaches:

$$u'(t) = -\nabla J(u(t))$$

- R. Hynd, E. Lindgren: *Approximation of the least Rayleigh quotient for degree p homogeneous functionals.* Journal of Functional Analysis 272 (2017)
- G. Gilboa: *Iterative Methods for Computing Eigenvectors of Nonlinear Operators*. Handbook of Mathematical Models and Algorithms in Computer Vision and Imaging (2021)
- L. Bungert, M. Burger, D. Tenbrinck: *Computing Nonlinear Eigenfunctions via Gradient Flow Extinction*

Inverse power methods:

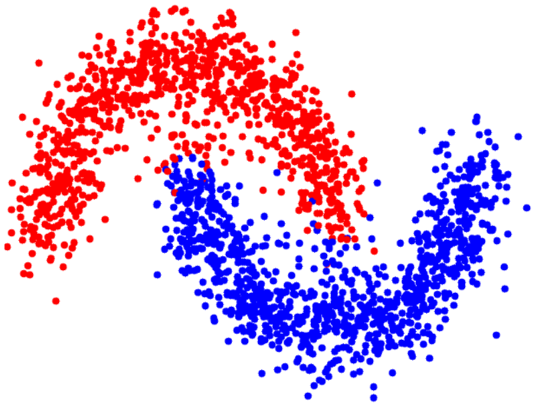
$$u^{k+1} = \frac{(\nabla J(u^k))^{-1}}{||(\nabla J(u^k))^{-1}||}$$

- M. Hein, T. Bühler: *An Inverse Power Method for Nonlinear Eigenproblems with Applications in 1-Spectral Clustering and Sparse PCA*. Advances in Neural Information Processing Systems 23 (2010)
- F. Bozorgnia: *Convergence of Inverse Power Method for First Eigenvalue of p-Laplace Operator*. Numerical Functional Analysis and Optimization 37 (2016)
- J. Laubmann, D. Tenbrinck, M. Friedrich: *Duality of Nonlinear Eigenproblems*. soon on ArXiv (2025)

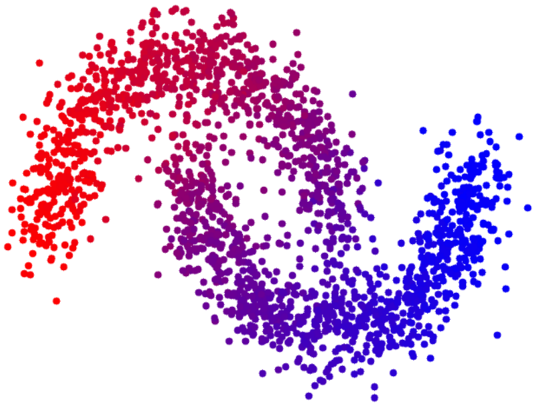
Nonlinear spectral clustering

In this setting one solves the following eigenvalue problem via inverse power iterations:

$$0 \in \Delta_1 u - \lambda \operatorname{sign}(u)$$



Eigenvector of 1-Laplacian



Eigenvector of 2-Laplacian

[9]Image: M. Hein, T. Bühler: *An Inverse Power Method for Nonlinear Eigenproblems with Applications in 1-Spectral Clustering and Sparse PCA*. Advances in Neural Information Processing Systems 23 (2010)

Nonlinear spectral image fusion

In this setting one has $J(u) = TV(u) = \int_\Omega ||\nabla u(x)|| \mathrm{d}x$ and solves the eigenvalue problem via gradient flow:

$$\lambda u \in \partial J(u)$$

[10]Image: M. Benning, M. Möller, R. Nossek, M. Burger, D. Cremers, G. Gilboa, C. Schönlieb: *Nonlinear Spectral Image Fusion*. SSVM (2017)

Outline

Weighted graphs in data science and machine learning

- Data modeling with graphs
- Differential operators on graphs

Eigenvalue problems on graphs

- Linear eigenvalue problems
- Nonlinear eigenvalue problems

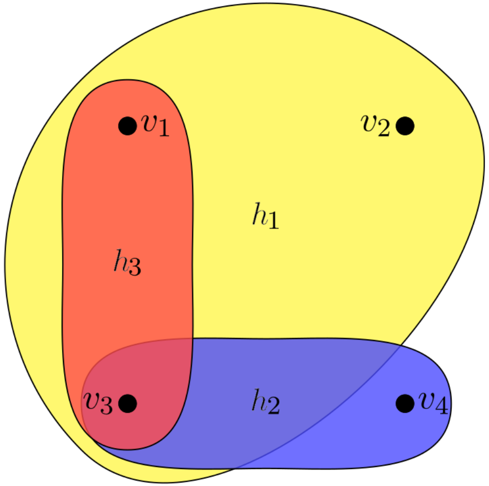
Extension to hypergraphs

- Data modeling with hypergraphs
- Differential operators on hypergraphs
- Eigenvalue problems on hypergraphs

Discrete data modeling using weighted hypergraphs

Idea:

Directly extend (traditional) weighted graphs by allowing more vertices in edges instead of only pairs of vertices.



[11] X. Ouvrard: *Hypergraphs: an introduction and review.* arXiv (2020)

Discrete data modeling using weighted hypergraphs

Idea:
Directly extend (traditional) weighted graphs by allowing more vertices in edges instead of only pairs of vertices.

Applications fields:

- Molecular chemistry

  J. Jost, R. Mulas: *Hypergraph Laplace operators for chemical reaction networks*. Advances in Mathematics 351 (2019)

- Social network analysis

  A. Fazeny, D. Tenbrinck, M. Burger: *Hypergraph p-Laplacians, Scale Spaces, and Information Flow in Networks*. SSVM (2023)

- Machine Learning

  Y. Feng, H. You, Z. Zhang, R. Ji, Y. Gao: *Hypergraph Neural Networks*. AAAI Conference (2019)

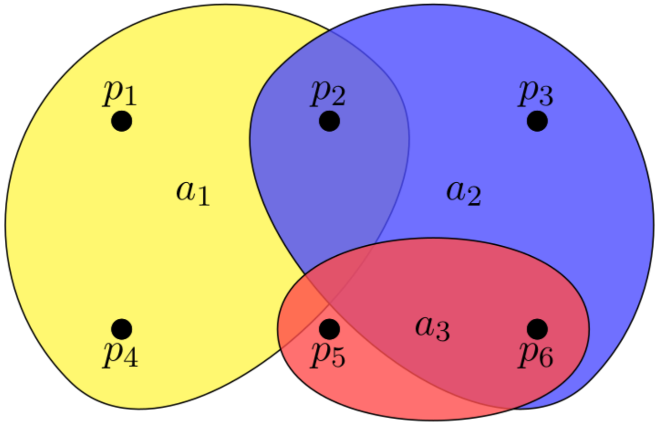
[11] X. Ouvrard: *Hypergraphs: an introduction and review*. arXiv (2020)

Hypergraphs for modeling co-authorship in papers

Example:
Set of scientific papers $P := \{p_1, \ldots, p_6\}$ and set of authors $A := \{a_1, a_2, a_3\}$. Possible modeling with hypergraphs:

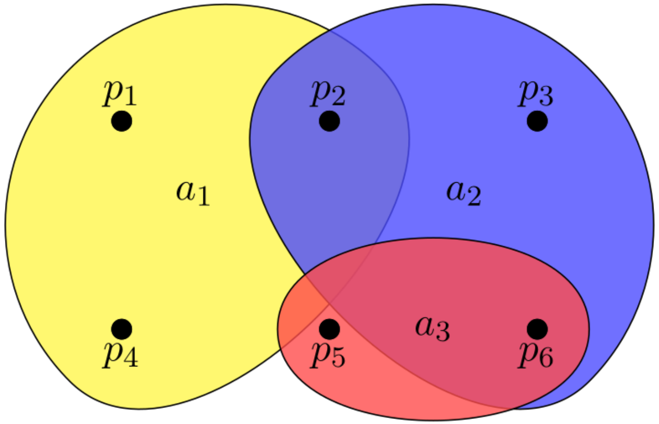
Hypergraphs for modeling co-authorship in papers

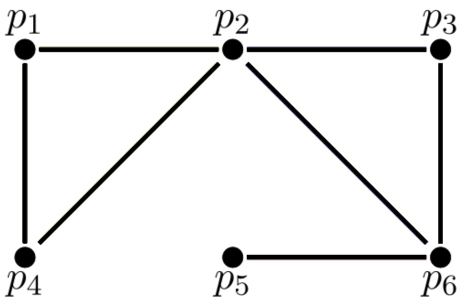
Example:
Set of scientific papers $P \coloneqq \{p_1, \ldots, p_6\}$ and set of authors $A \coloneqq \{a_1, a_2, a_3\}$. Possible modeling with hypergraphs:



Hypergraphs for modeling co-authorship in papers

Example:
Set of scientific papers $P := \{p_1, \ldots, p_6\}$ and set of authors $A := \{a_1, a_2, a_3\}$. Possible modeling with hypergraphs:

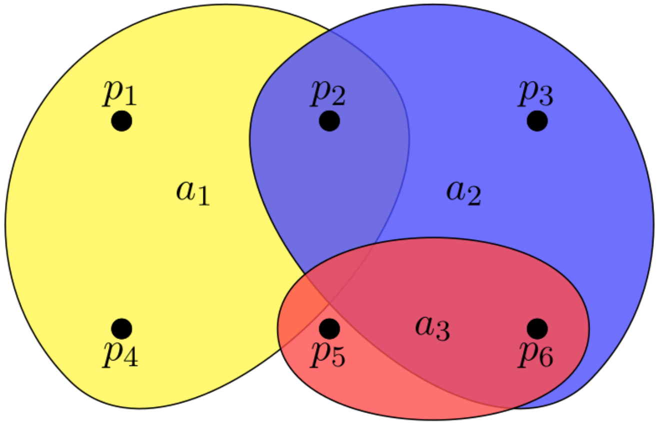


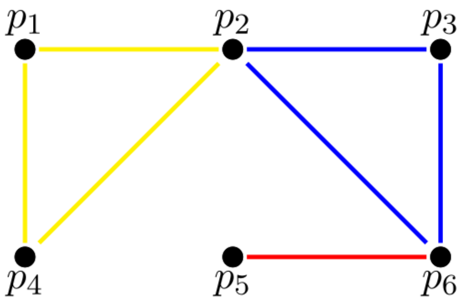


Hypergraphs for modeling co-authorship in papers

Example:

Set of scientific papers $P \coloneqq \{p_1, \ldots, p_6\}$ and set of authors $A \coloneqq \{a_1, a_2, a_3\}$. Possible modeling with hypergraphs:

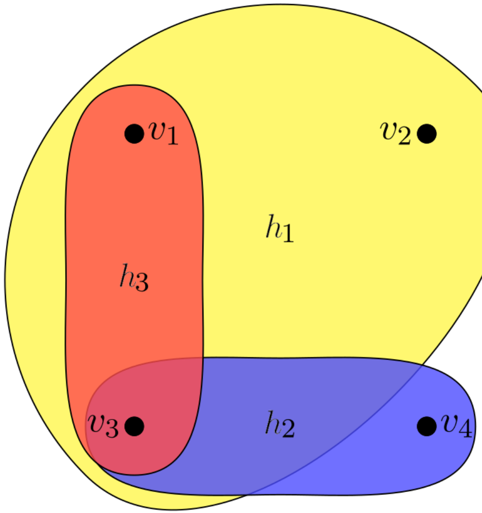




Finite weighted hypergraphs

A finite weighted hypergraph $HG = (V, H, w)$ consists of:

- a finite set of vertices $V = (v_1, \ldots, v_n)$
- a finite set of hyperedges $H \subset \mathcal{P}(V) \setminus \emptyset$
- a weight function $w\colon H \to [0, 1]$ with: $w(h) > 0 \Leftrightarrow h \in H$



[12] Image: C. Schiller: *The PageRank Algorithm on Graphs and Hypergraphs.* Bachelor's thesis at FAU (2024)

Differential operators on hypergraphs

Observation:

There are several possibilities to generalize the weighted graph gradient operator, e.g., sum, min/max, mean, random, . . .

[13] A. Fazeny, D. Tenbrinck, K. Lukin, M. Burger: Hypergraph p-Laplacians and Scale Spaces. JMIV 66. p. 529-549 (2024)

[14] J. Jost, R. Mulas, D. Zhang: *p-Laplace Operators for Oriented Hypergraphs*. Vietnam Journal of Mathematics Oct (2021)

Differential operators on hypergraphs

Observation:
There are several possibilities to generalize the weighted graph gradient operator, e.g., sum, min/max, mean, random, . . .

Let $h \in H$ be a hyperedge and $u \in h$ be a **chosen vertex** within h. Then the hypergraph gradient operator is defined as:

$$\nabla^u f(h) = \nabla f(u; h) := \sqrt{w(h)} \sum_{v_i \in h} (f(v_i) - f(u)) \tag{7}$$

[13] A. Fazeny, D. Tenbrinck, K. Lukin, M. Burger: Hypergraph p-Laplacians and Scale Spaces. JMIV 66. p. 529-549 (2024)
[14] J. Jost, R. Mulas, D. Zhang: *p-Laplace Operators for Oriented Hypergraphs*. Vietnam Journal of Mathematics Oct (2021)

Differential operators on hypergraphs

Observation:
There are several possibilities to generalize the weighted graph gradient operator, e.g., sum, min/max, mean, random, ...

Let $h \in H$ be a hyperedge and $u \in h$ be a **chosen vertex** within h. Then the hypergraph gradient operator is defined as:

$$\nabla^u f(h) \;=\; \nabla f(u;h) \;:=\; \sqrt{w(h)} \sum_{v_i \in h} (f(v_i) - f(u)) \tag{7}$$

Analogously to weighted graphs, we deduce the hypergraph p-Laplace operator[14] as:

$$\Delta_p f(u) \;=\; \operatorname{div}\left(|\nabla^u f|^{p-2} \nabla^u f\right)(u) \tag{8}$$

[13] A. Fazeny, D. Tenbrinck, K. Lukin, M. Burger: Hypergraph p-Laplacians and Scale Spaces. JMIV 66. p. 529-549 (2024)
[14] J. Jost, R. Mulas, D. Zhang: *p-Laplace Operators for Oriented Hypergraphs*. Vietnam Journal of Mathematics Oct (2021)

Eigenvalue problems on hypergraphs

Aim:
Formulate eigenvalue problems on hypergraphs, e.g., the linear case of $p = 2$:

Find a vertex function $f\colon V \to \mathbb{R}$ such that for all $u \in V$ we have:

$$\lambda f(u) \;=\; \Delta f(u) \tag{9}$$

Eigenvalue problems on hypergraphs

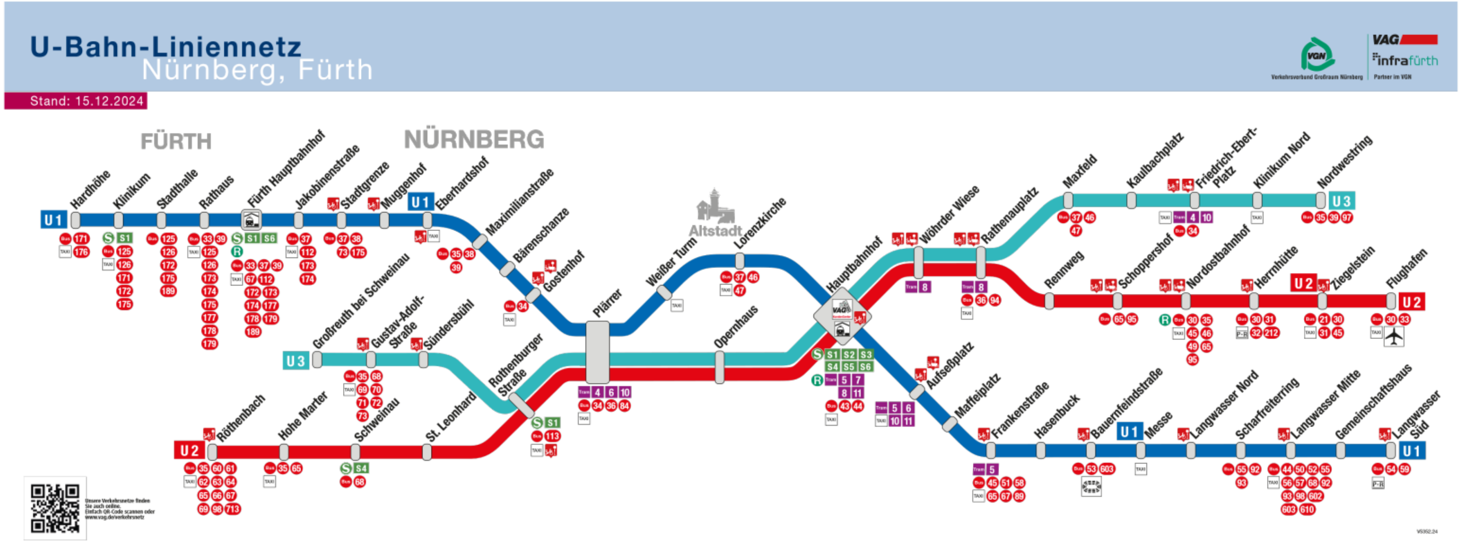
Aim:
Formulate eigenvalue problems on hypergraphs, e.g., the linear case of $p = 2$:

Find a vertex function $f : V \to \mathbb{R}$ such that for all $u \in V$ we have:

$$\lambda f(u) = \Delta f(u) = \operatorname{div}(\nabla^u f)(u) = \sum_{\substack{h \in H \\ u \in h}} w(h) \sum_{v_i \in h} (f(v_i) - f(u)) \tag{9}$$

Eigenvalue problems on hypergraphs

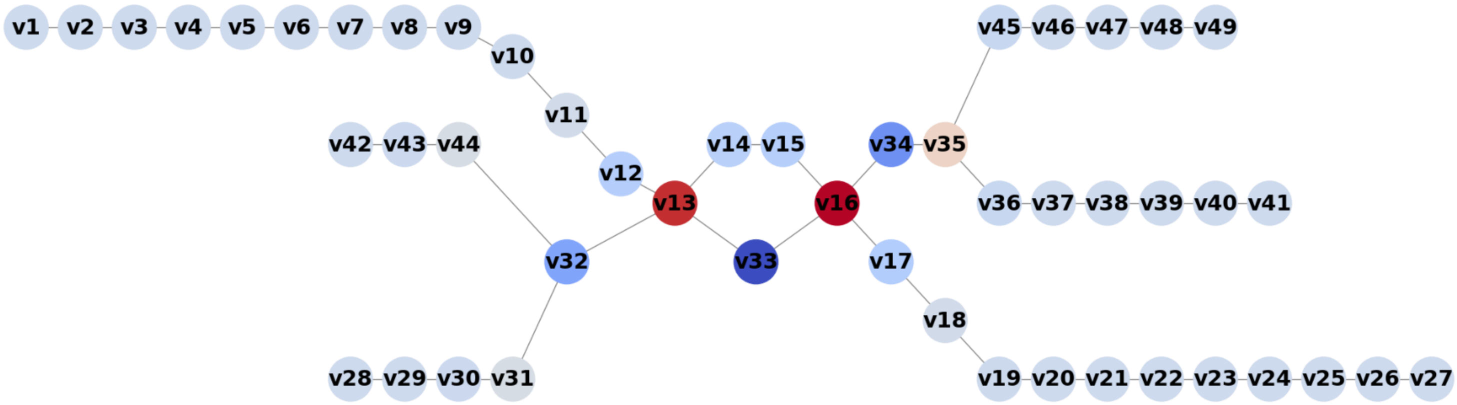
Example: PageRank[15] on subway stations in Nürnberg



[15] Y. Takai, A. Miyauchi, M. Ikeda, Y. Yoshida: *Hypergraph clustering based on PageRank*. Proceedings of KDD. p.1970-1978 (2020)
[16] T. Strauß: The eigenvalue problem on hypergraphs. Bachelor's thesis at FAU (2025)

Eigenvalue problems on hypergraphs

Example: PageRank[15] on subway stations in Nürnberg



[15] Y. Takai, A. Miyauchi, M. Ikeda, Y. Yoshida: *Hypergraph clustering based on PageRank*. Proceedings of KDD. p.1970-1978 (2020)

[16] T. Strauß: The eigenvalue problem on hypergraphs. Bachelor's thesis at FAU (2025)

Conclusion

Take-away message:

- Weighted graphs are a universal tool for data modeling
- Hypergraphs extend graphs and allow for additional information
- (Nonlinear) eigenvalue problems are used for various applications in machine learning and data science
- Next goal: Investigate nonlinear eigenvalue problems on hypergraphs

Thank you for your attention!

Kudos to my collaborators:


M. Burger


A. Fazeny


L. Bungert


M. Friedrich


J. Laubmann

Any questions?