Applications of AAA Rational Approximation

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r = aaa(f(Z),Z)
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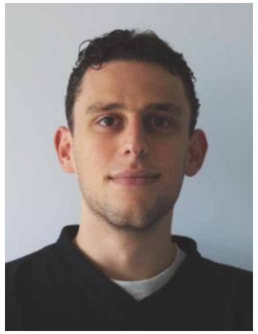
Nakatsukasa-Sète-T., *SISC* 2018. "AAA" originates from "Adaptive Antoulas-Anderson".

Yuji Nakatsukasa
Oxford

Olivier Sète
Berlin

Stefano Costa
Piacenza



Daan Huybrechs
Leuven

It all started in Autumn 2016.

AAA collaborators

Peter Baddoo
MIT

Toby Driscoll
Delaware

André Weideman
Stellenbosch

Abi Gopal
Yale

AAA	Nakatsukasa-Sète-T., *SISC* 2018
AAA-Lawson	Nakatsukasa-T., *SISC* 2020
Conformal mapping	Gopal-T., *Numer. Math.* 2019 & *CMFT* 2020
Periodic AAA	Baddoo, *SISC* 2021
Exponential clustering	T.-Nakatsukasa-Weideman, *Numer. Math.* 2021
Equispaced interpolation	Huybrechs-T., *BIT Numer. Math.* to appear
AAA-least-squares	Costa-T., *Proc. 8ECM* to appear
Analytic continuation	T., *JJIAM* in preparation
Adaptive sampling	Driscoll-T., in preparation

AAA algorithm

Sample points $\zeta_j \in \mathbb{C}$ with data values $f(\zeta_j)$

Barycentric representation of the rational approximation:

$$r(z) = \frac{n(z)}{d(z)} = \sum_{k=1}^{m} \frac{a_k f(z_k)}{z - z_k} \Bigg/ \sum_{k=1}^{m} \frac{a_k}{z - z_k}$$

← barycentric weights $\{a_k\}$

← support points $\{z_k\}$, with $r(z_k) = f(z_k)$

Algorithm:

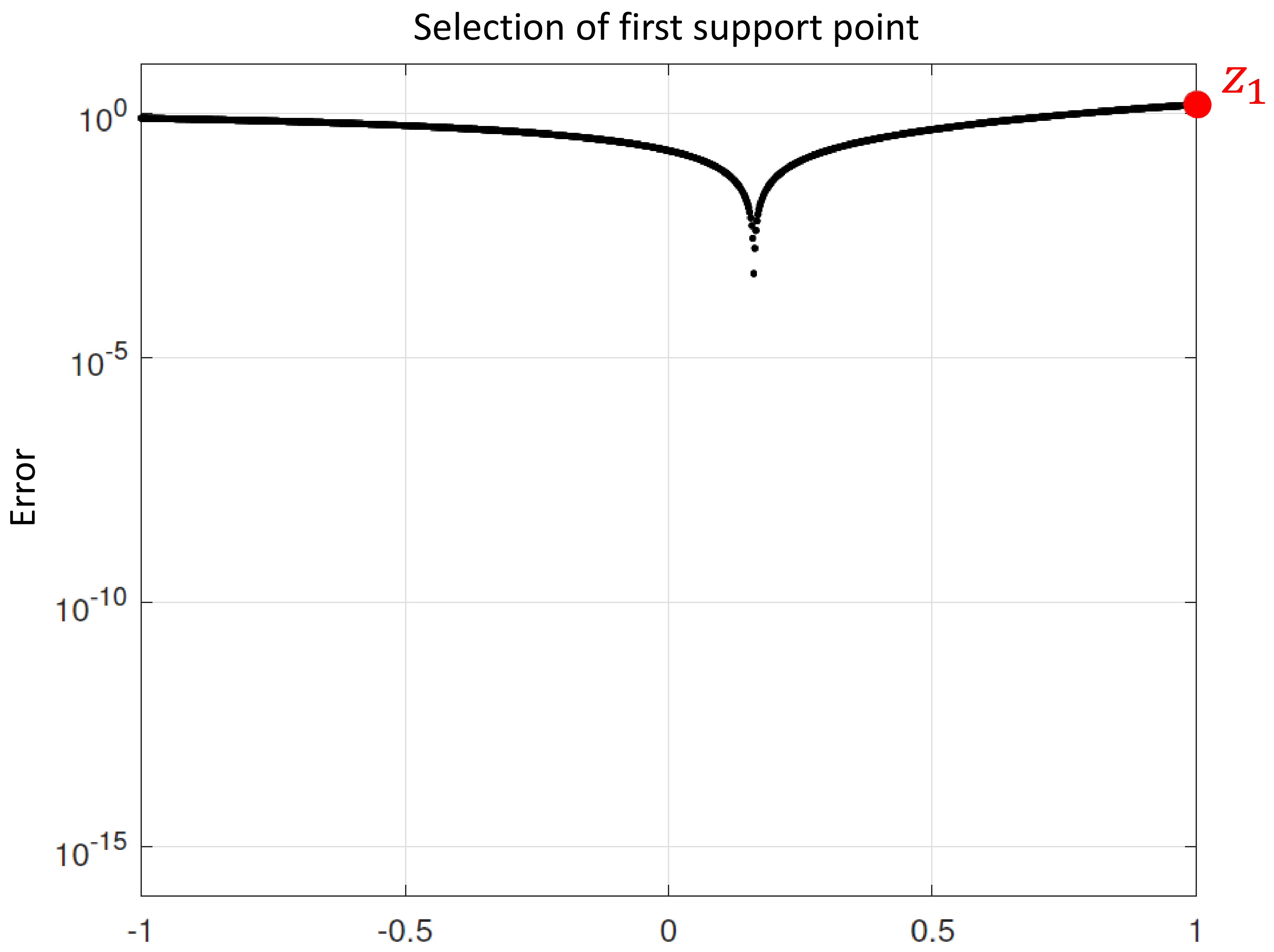
For $m = 1,2, \ldots$ until relative error $< 10^{-13}$:

- Support point z_m: sample point ζ_i where error $|f_i - r(\zeta_i)|$ is largest.
- Barycentric weights $\{a_k\}$: minimize linearized least-squares error $\| f d - n \|$.

← tall skinny SVD

AAA typically produces near-optimal approximations in a fraction of a second.

Example $f(z) = e^z$ on $[-1,1]$



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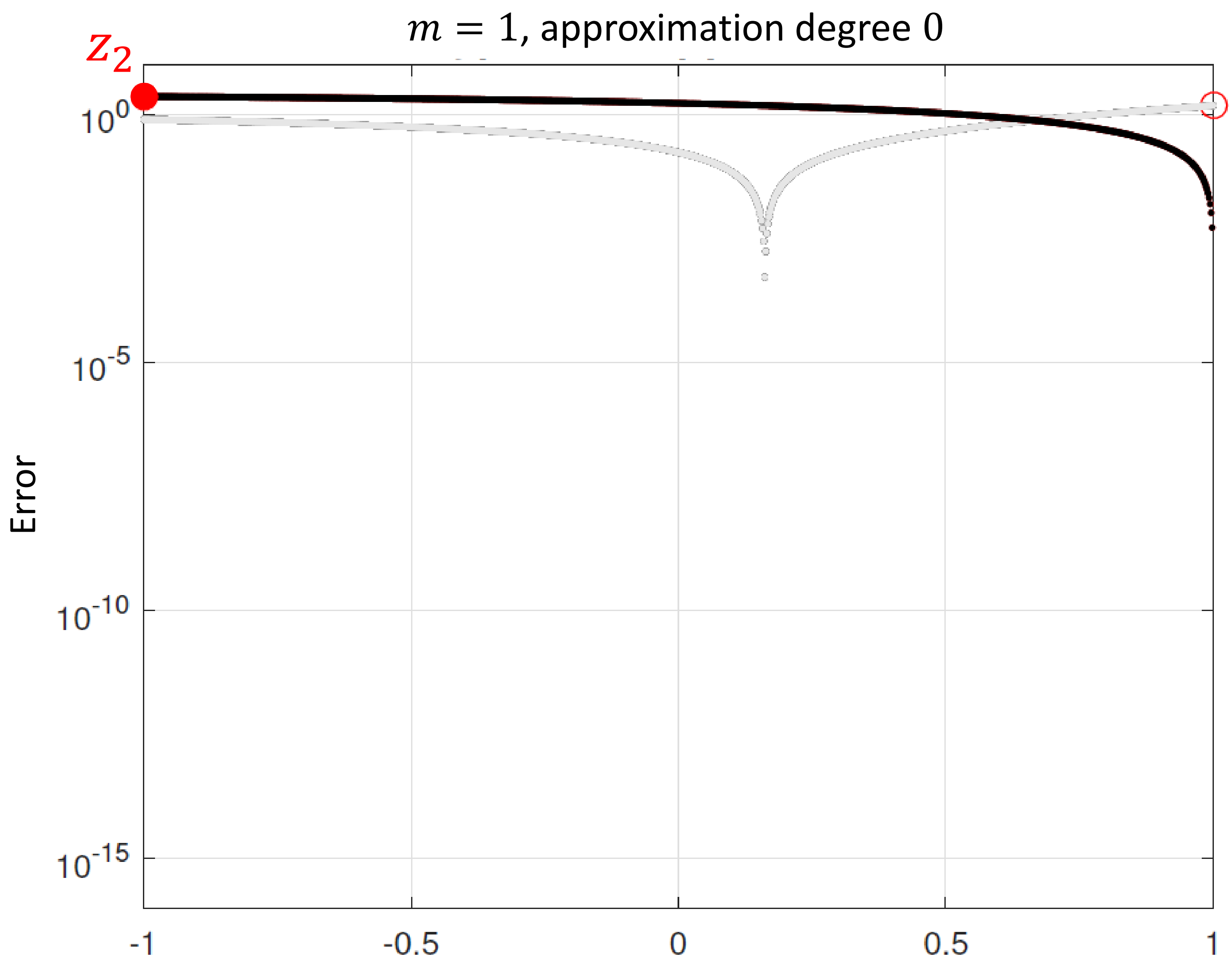

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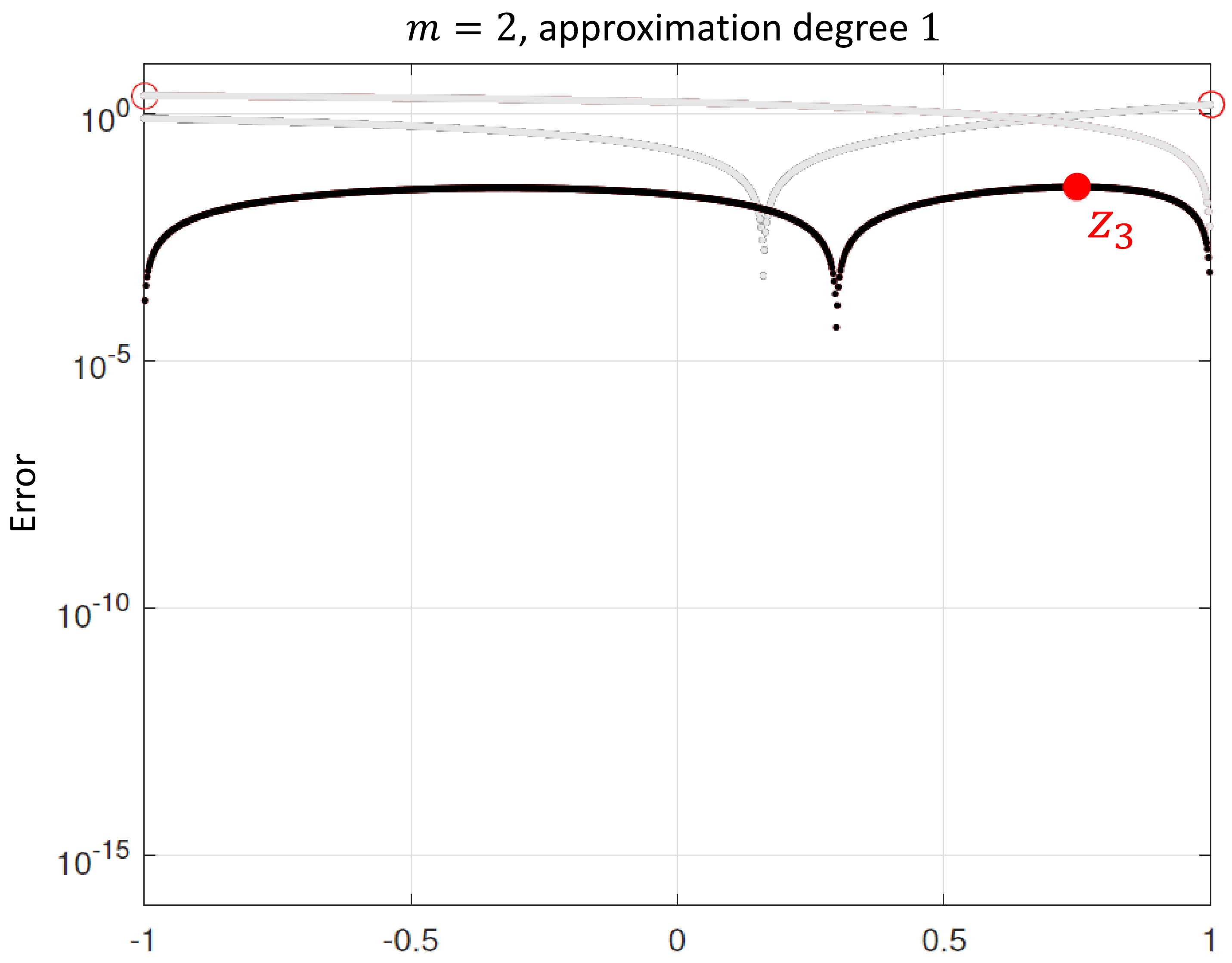

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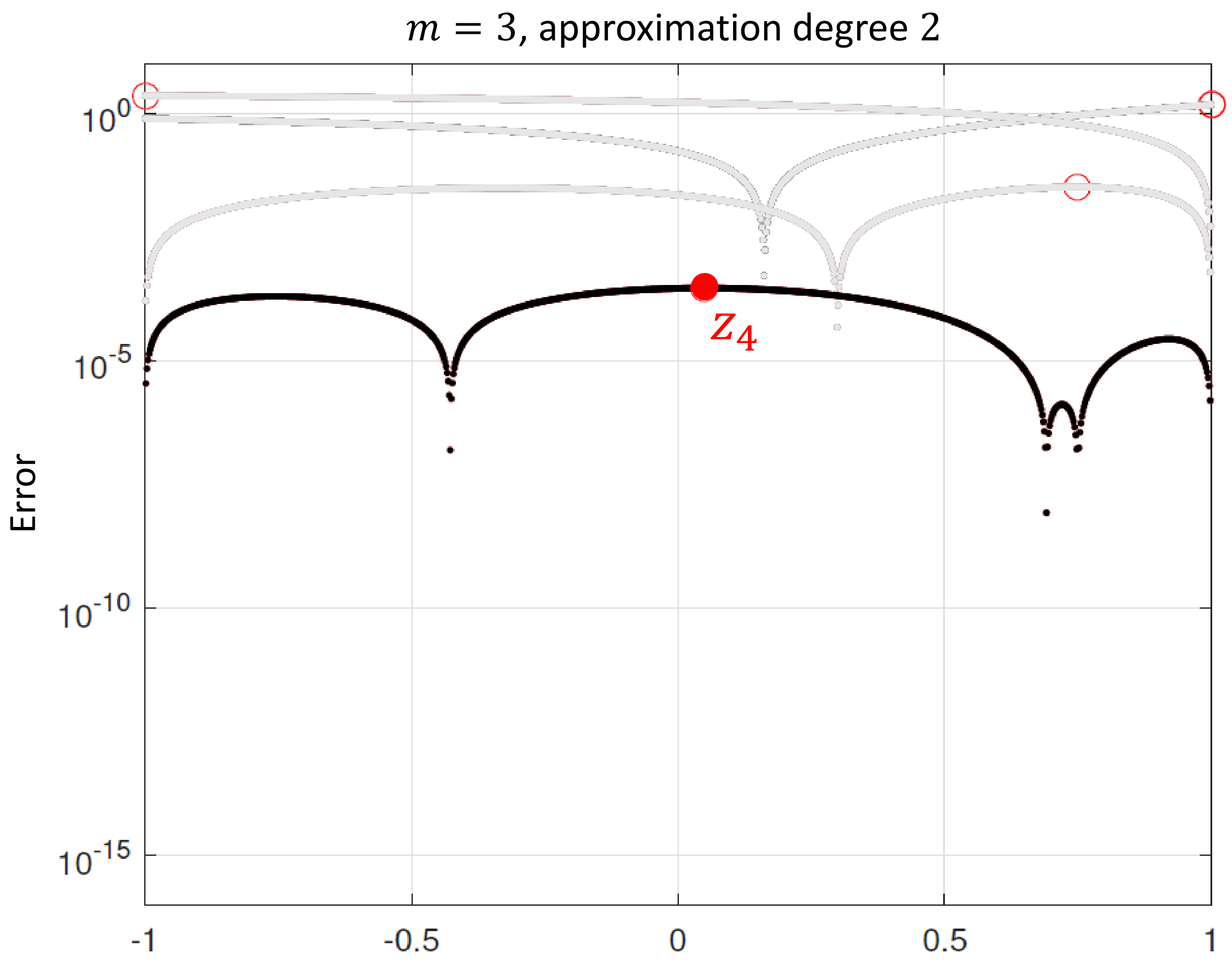

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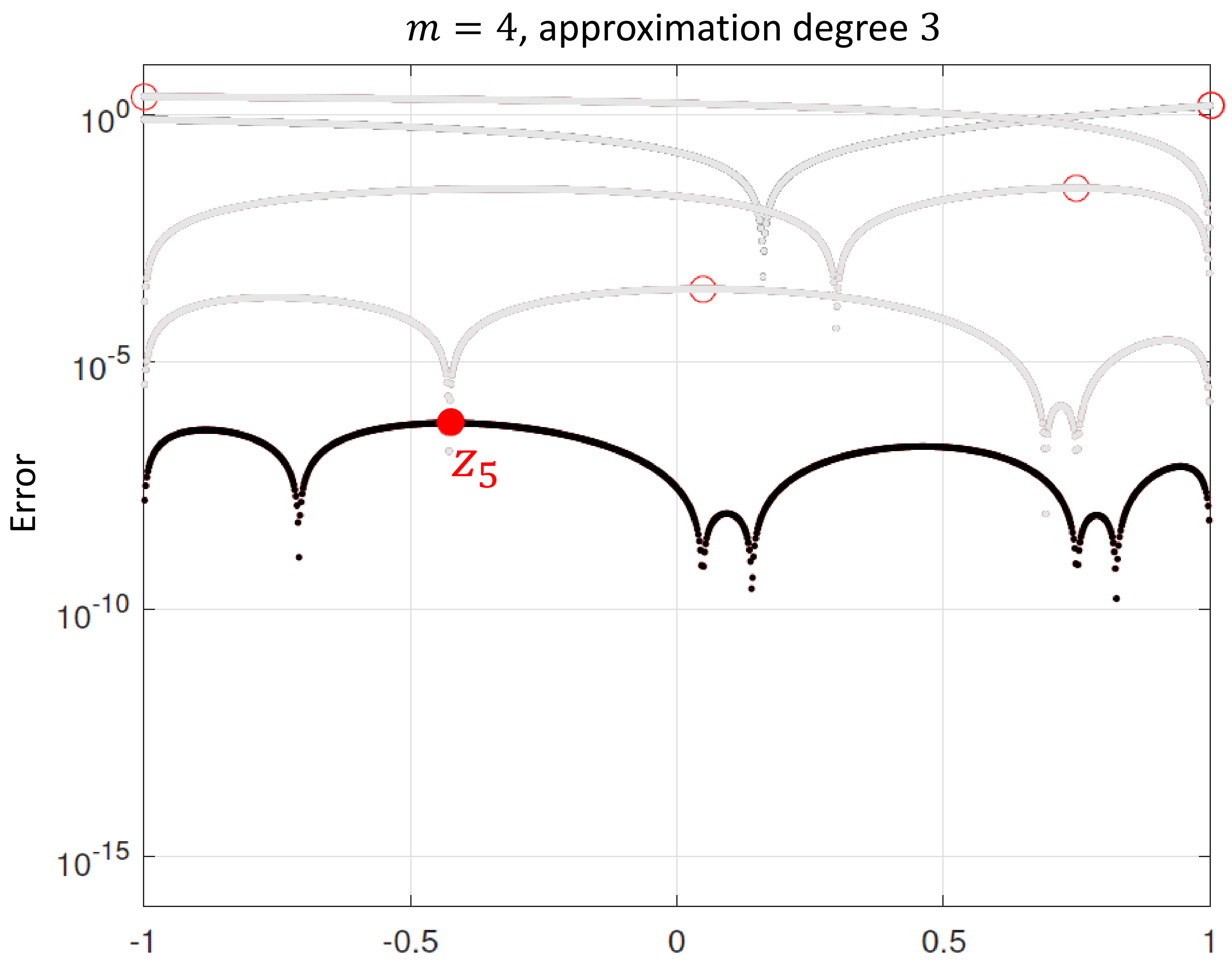

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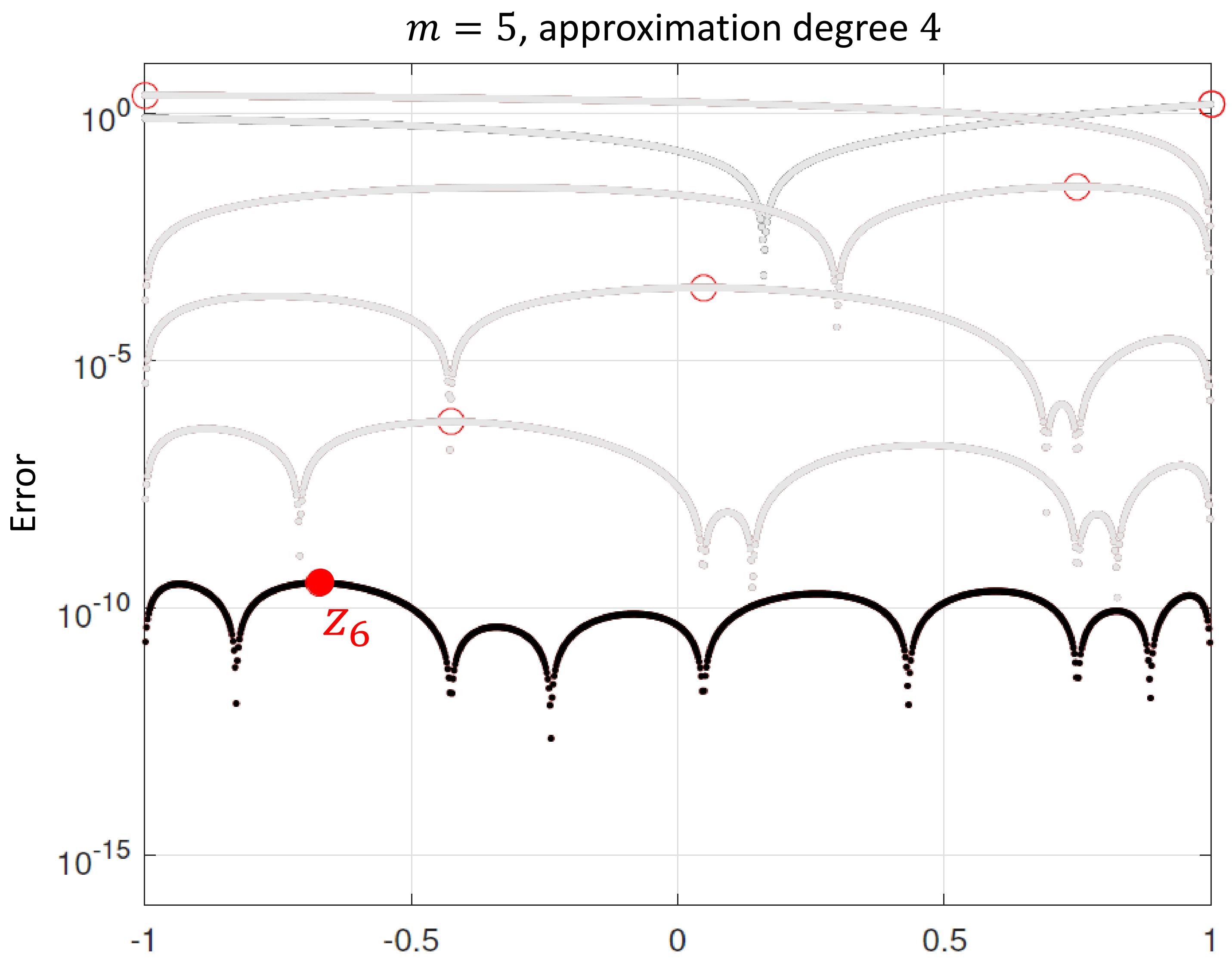
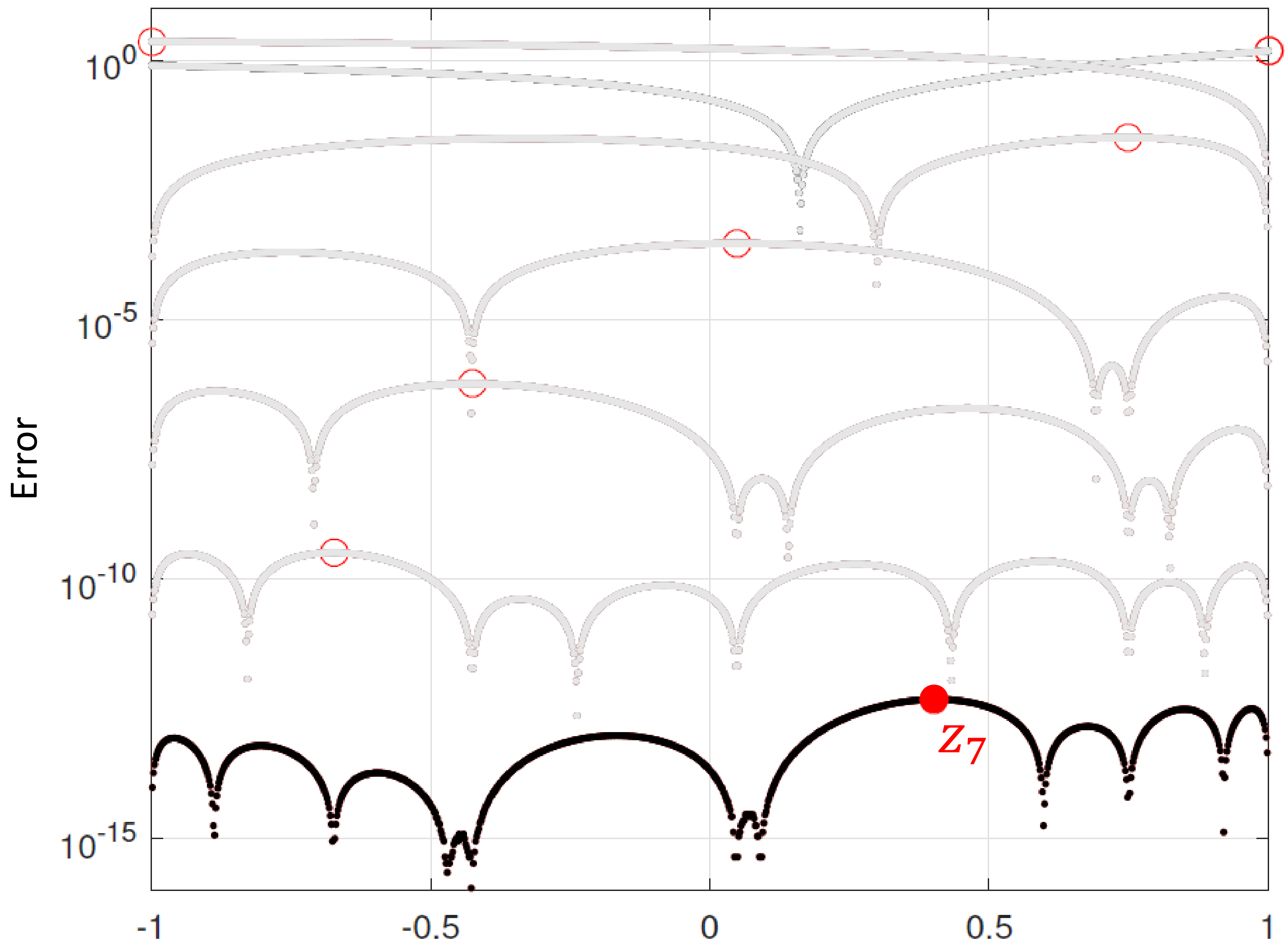


Example $f(z) = e^z$ on $[-1,1]$

$m = 6$, approximation degree 5



Why are rational functions useful?

Mathematically like polynomials, but with poles not constrained to lie at $z = \infty$.
Unlike polynomials, can approximate singular/near-singular functions and on unbounded domains.

Rational approximation theory: Walsh, Newman, Gonchar, Rakhmanov, Saff, Stahl,....

Applications in this talk

Data fitting	expz, equispaced, clustering
Analytic continuation	gammaplot, zetafun, lorenz, burgers, extension
Detection of singularities and branch cuts	twodisks, branchcut, splinenodes
Linear algebra/model order reduction	beam
Analytic/coanalytic splitting	wienerhopf
Laplace problems	conf, conf2, wave, Lshape

The mystery that drives me crazy

Before AAA, rational approximation was just another hard nonlinear optimization problem.
Now it's as if we have magical powers to solve it.
Are other problems truly harder? — like fitting by variable-width Gaussians? —
or have we just not yet found their magic algorithms?