Hybrid Modeling and System Identification: Past and Future Directions

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https://helonayala.github.io

FAU MoD Lecture

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PUCPR

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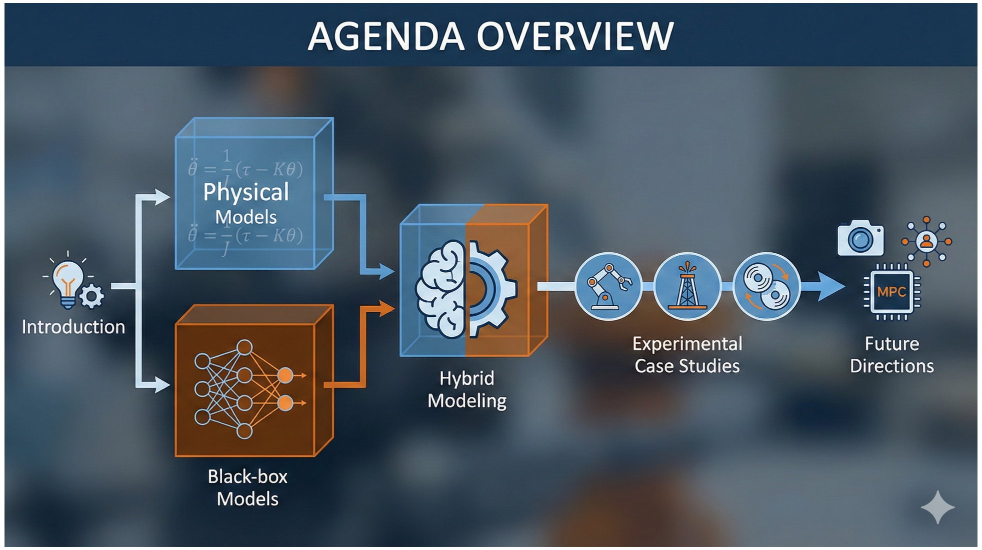
Friedrich-Alexander-Universität
DYNAMICS, CONTROL, MACHINE LEARNING AND NUMERICS

CNPq

erc

Agenda

1. Preliminaries
   - System Identification
   - Hybrid Modeling Paradigms
2. Prior work
   - Drill-string
   - Flexible-joint manipulator
3. Future work
   - Hybrid multimodal model learning for approximate MPC

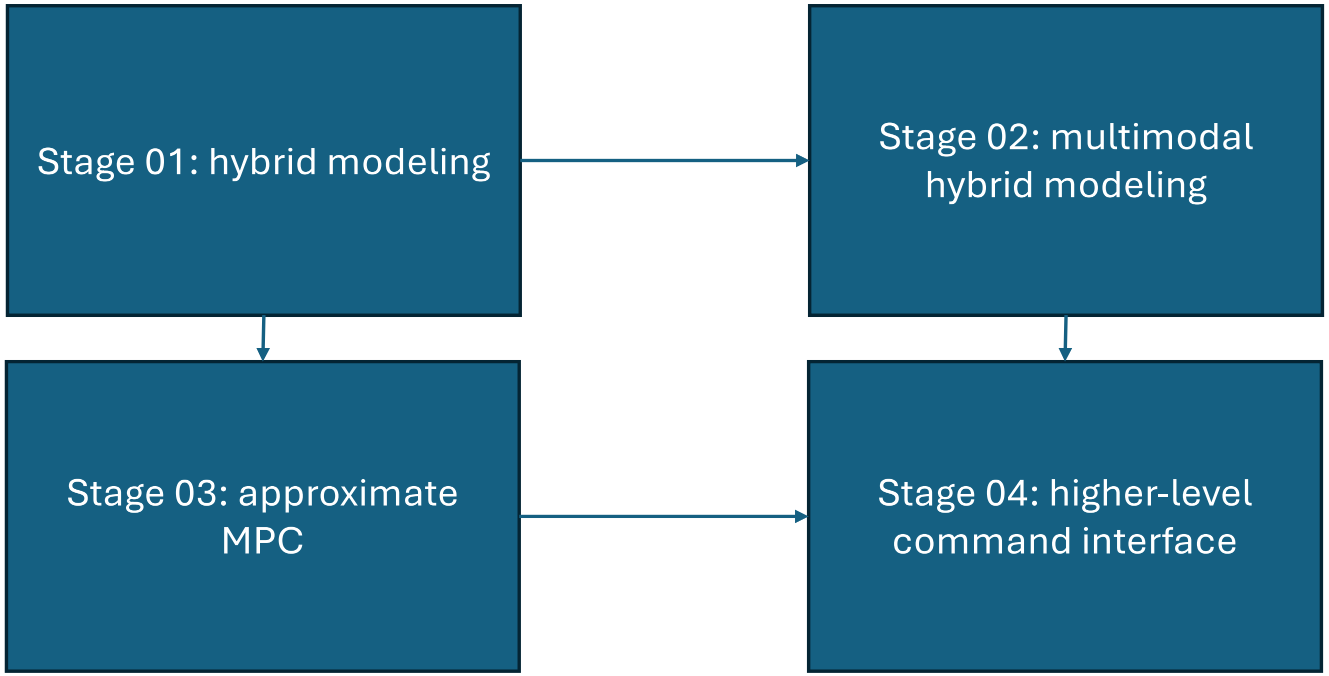


Introduction

- Since 2023 at PUCPR
  - 2025-2027 ERC/CNPq project
- Prior positions:
  ZF (Germany), Embraer, IBM Research, PUC-Rio
- Research interests:
  system identification, advanced controls, and applied ML

ERC/CNPq Project Goals

Hybrid system identification for approximate MPC



Former students

8 PhDs, 19 MScs, 28 BScs

Profs. Elias/Daniel
(Military Institute of Engineering)

Pedro Calderano
(visiting PhD @ Yale)

Mateus
(PhD @ Stanford)

Fernanda
(IBM)

Ingrid
(Kongsberg Maritime)

Felipe / Pedro / Carlos
(Petrobras)

Louise Erthal
(Technip FMC)

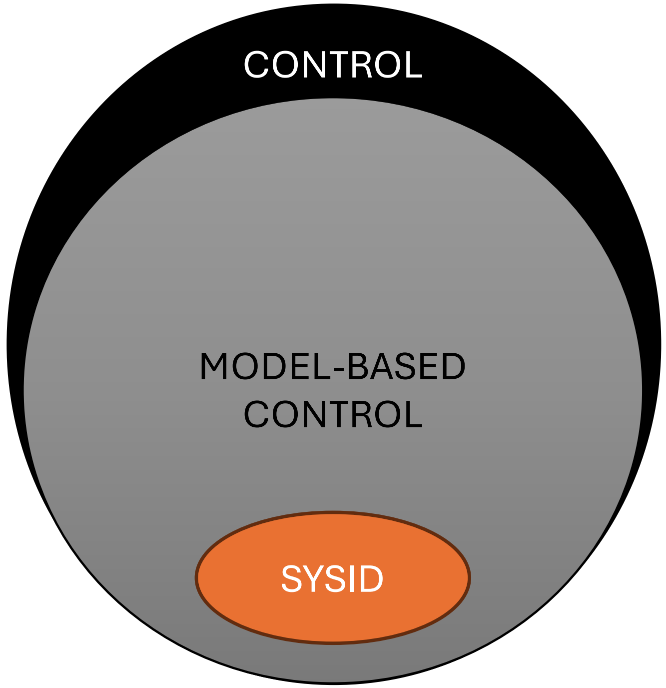
Iron
(Volvo Trucks)

Lucas (Boeing)

Pablo
(Embraer)

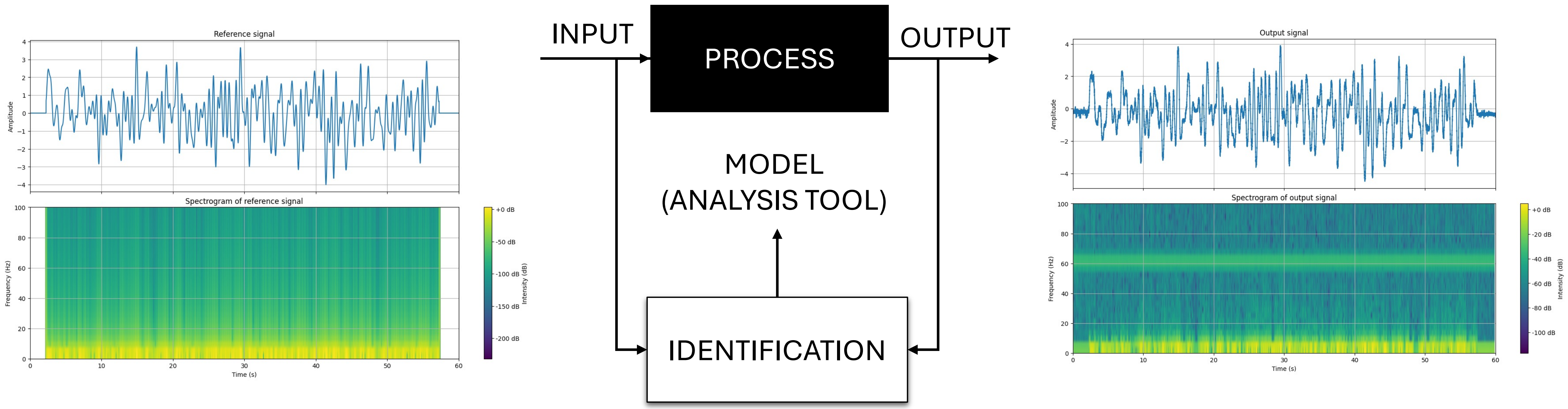
Daniel
(Tik Tok)

System Identification (SYSID)



What is system identification?

Identification is the task of using input-output data to build a model: a mathematical abstraction of the process [2]



[2] Arun K Tangirala.
Principles of system identification: Theory and practice.
CRC Press, 2014.

Example 01: a 2WIP mobile robot

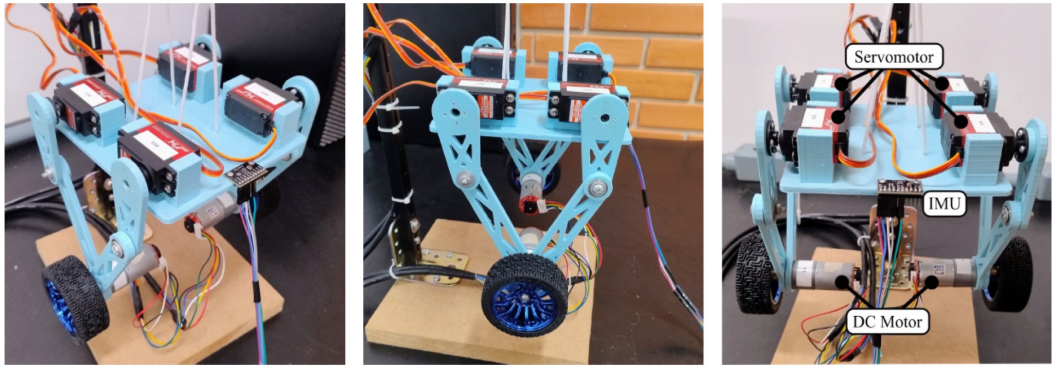


Figure 1. Overview (left and center) and main components (right) of the robot prototype on the test bench.

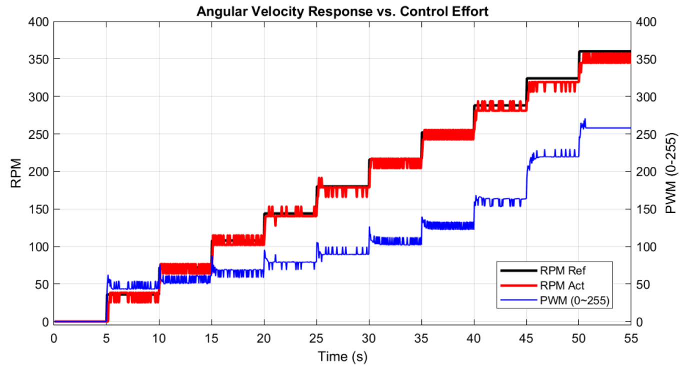


Figure 7. Speed control validation. The plot compares the motor's response (RPM Measured) with the step reference (RPM Reference). The control signal (PWM Feedback) is also included to assess the system's actuation effort.

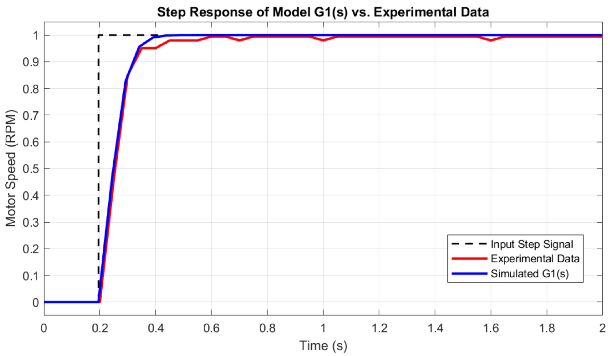


Figure 4. Step input of PWM and open-loop motor speed (RPM) responses.

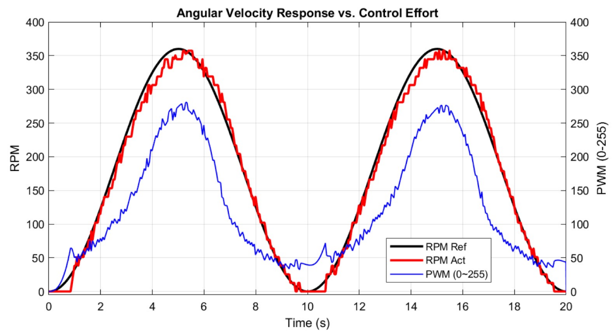
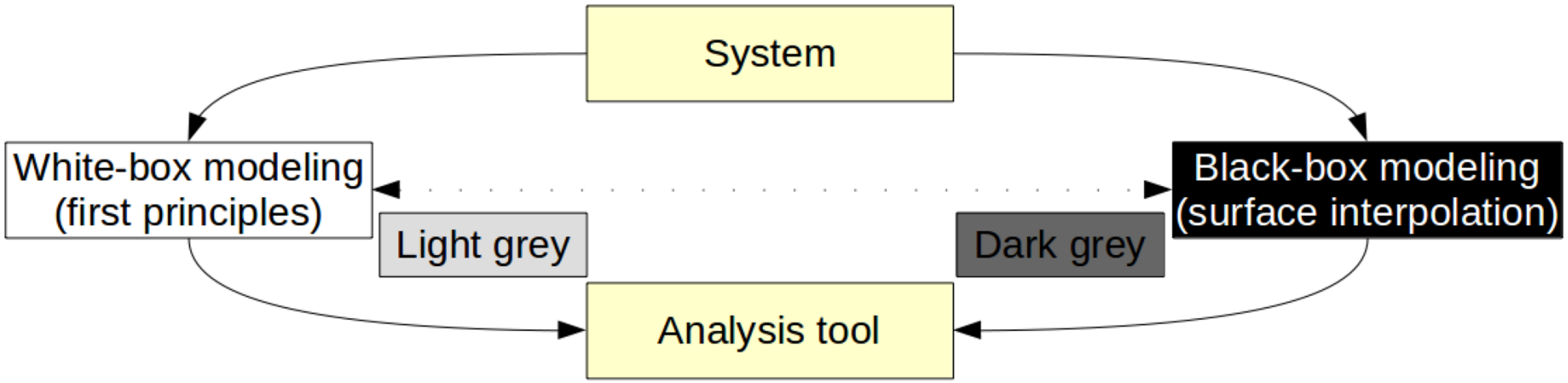


Figure 8. Closed-loop response to a sinusoidal trajectory. The left axis is motor speed (RPM); the right axis is the control signal (PWM). This dual-axis plot enables a direct comparison between system output and control effort.

Paradigms in SYSID



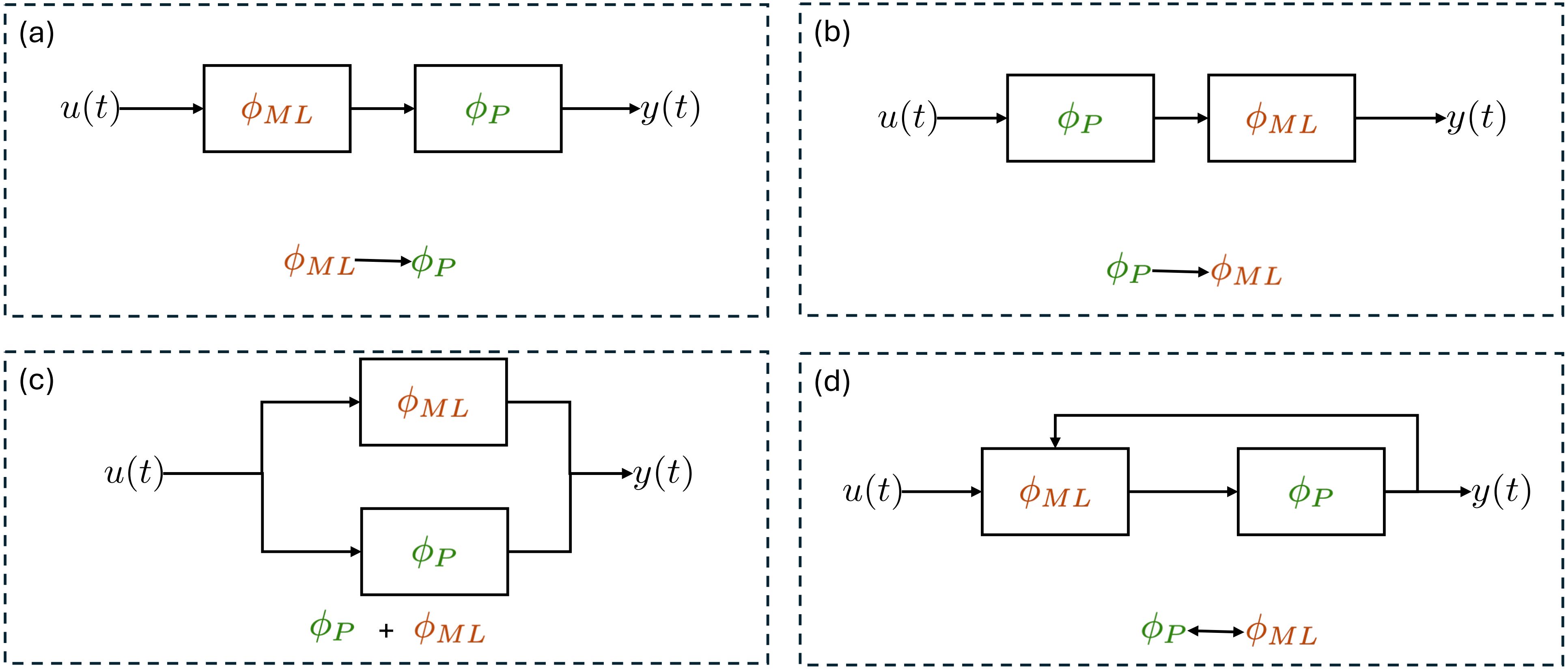
- What's the best modeling approach?
  - Prof. Ljung: "Our acceptance of models should be guided by **usefulness** rather than **truth**"

Typical discrete-time model structures

	LINEAR	**NONLINEAR**
State-space	$\begin{cases} x[k+1] = Ax[k] + Bu[k] \\ y[k] = Cx[k] + Du[k] \end{cases}$	$\begin{cases} x[k+1] = f(x[k], u[k], \theta) \\ y[k] = g(x[k], u[k], \theta) \end{cases}$
Difference equation	$y[k] + \sum_{i=1}^{n_a} a_i y[k-i] = \sum_{j=1}^{n_b} b_j u[k-j]$	$\begin{aligned} y[k] = F(&y[k-1], \dots, y[k-n_y], \\ &u[k-1], \dots, u[k-n_u], \theta) \end{aligned}$

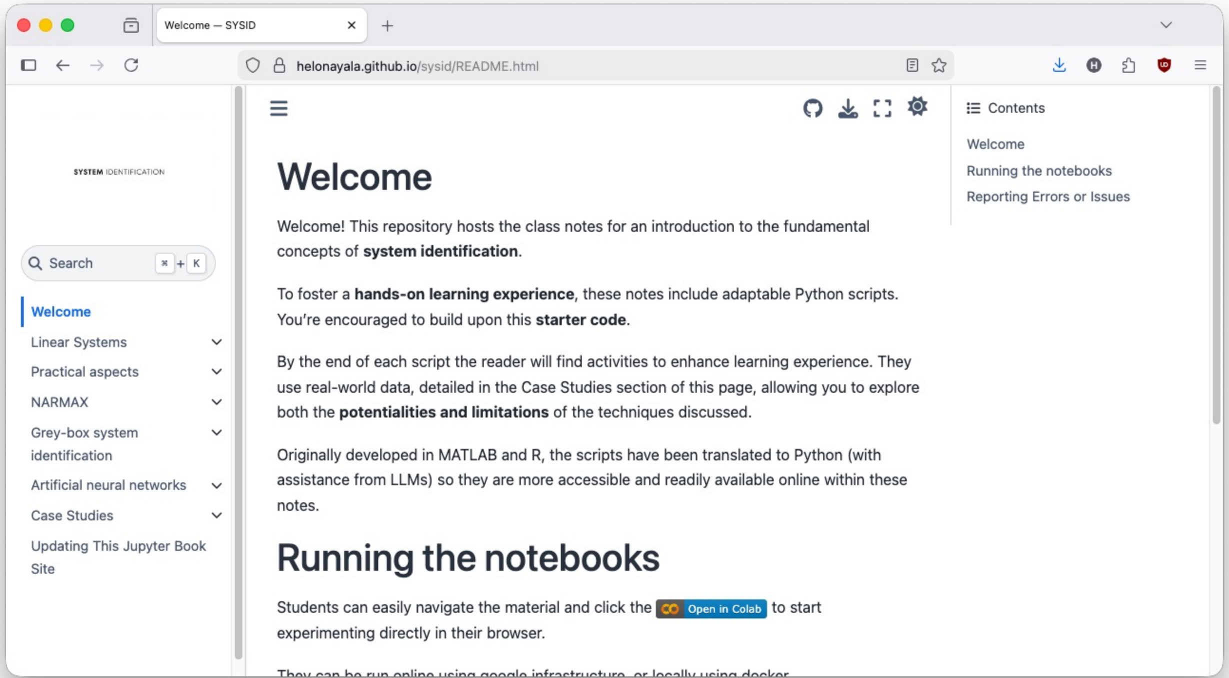
Following standardized model structures allows using model-based control approaches

Hybrid modeling paradigms



Mukherjee and Zavala, **Physics-constrained machine learning for chemical engineering.** *Current Opinion in Chemical Engineering* 2026, 51:101228

SYSID course online



https://helonayala.github.io/sysid/

Prior work

Mechanical Systems and Signal Processing 186 (2023) 109815

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Nonlinear ensemble gray and black-box system identification of friction induced vibrations in slender rotating structures

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Goals:

- Build physics-based and black-box models
- Improve results by merging approaches

Rotating structure subject to friction

A torque is produced by the electric motor

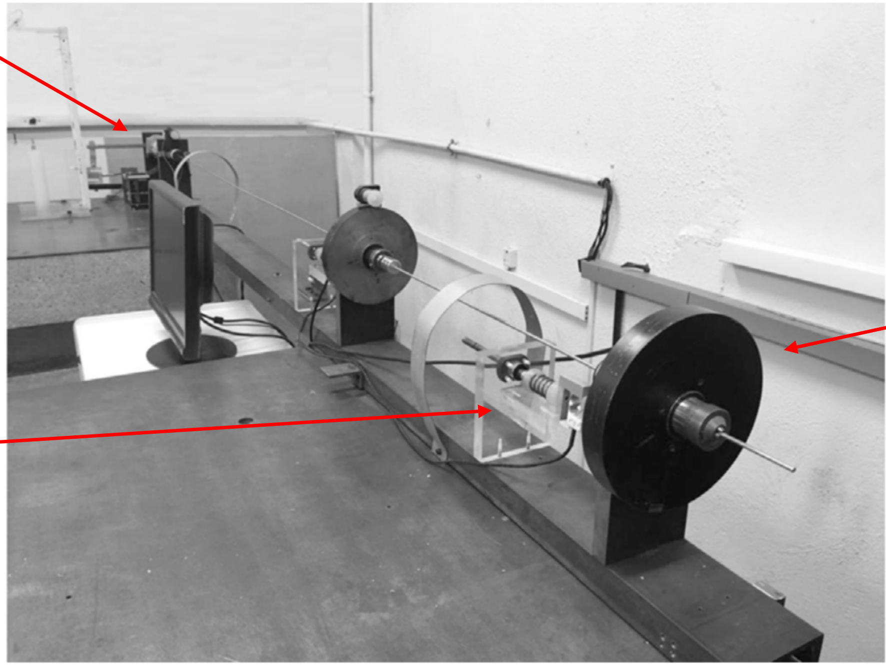

Fig. 2. Experimental test rig.

Resistive torque is controlled (manually) to induce oscilations in the structure – unknown for modeling purposes

Velocities are measured in order to build the dynamical model based on data
(and some prior knowledge)

Physical model

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1 & -1 \\ -k/J_d & -(c+c_d)/J_d & c/J_d \\ k/J_m & c/J_m & -(c+c_m)/J_m \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\theta}_d \\ \dot{\theta}_m \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1/J_d & 0 \\ 0 & 1/J_m \end{bmatrix} \begin{bmatrix} T_f \\ \tau_m \end{bmatrix}$$

$$y_m = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\theta}_d \\ \dot{\theta}_m \end{bmatrix};$$

The interest of the present analysis is to estimate the mechanical parameters of the test rig, shaft stiffness and damping, and friction parameters. These unknown parameters are organized in the vector β as follows:

$$\beta_1 = \begin{bmatrix} k & c & c_d & c_m & T_C \end{bmatrix}, \qquad \beta_2 = \begin{bmatrix} k & c & c_d & c_m & T_C & T_S & V_S \end{bmatrix},$$
$$\beta_3 = \begin{bmatrix} k & c & c_d & c_m & T_C & \sigma_0 \end{bmatrix}, \qquad \beta_4 = \begin{bmatrix} k & c & c_d & c_m & T_C & T_S & \alpha_1 & \alpha_2 \end{bmatrix},$$

subscripts 1, 2, 3 and 4 correspond to the optimization problems with the four friction models: Coulomb, Stribeck, Dahl, and Stefanski et al. respectively. For the optimization problem, lower and upper bounds for the unknown parameters were defined by physical restrictions.

Proposed approach

$$y_e(k) = \overbrace{f(x(k), u(k))}^{\text{gray-box}} + \overbrace{g(e(k-1), \ldots, e(k-n_e), u(k-1), \ldots, u(k-n_u))}^{\text{black-box}}$$

First principles' model

Complementary model (black-box) to model remaining dynamics in the data

Step-by-step
1- adjust gray-box model
2- analyse residual properties (are they random?)
3- build black-box model

Results: gray-box model

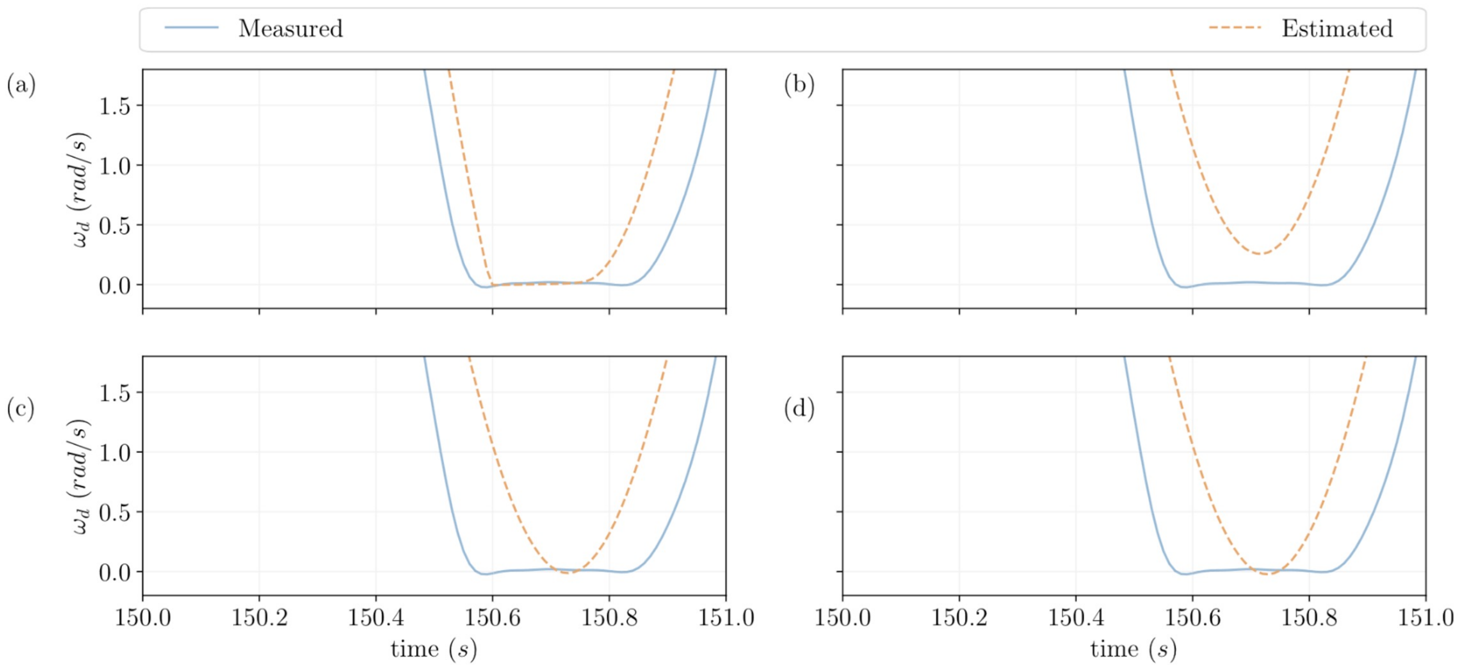

Fig. 5. Comparison of measured and predicted disc angular velocity using gray-box model based on (9), one stick phase interval: (a) Coulomb; (b) Stribeck; (c) Dahl; (d) Stefanski et al.

Results: hybrid model

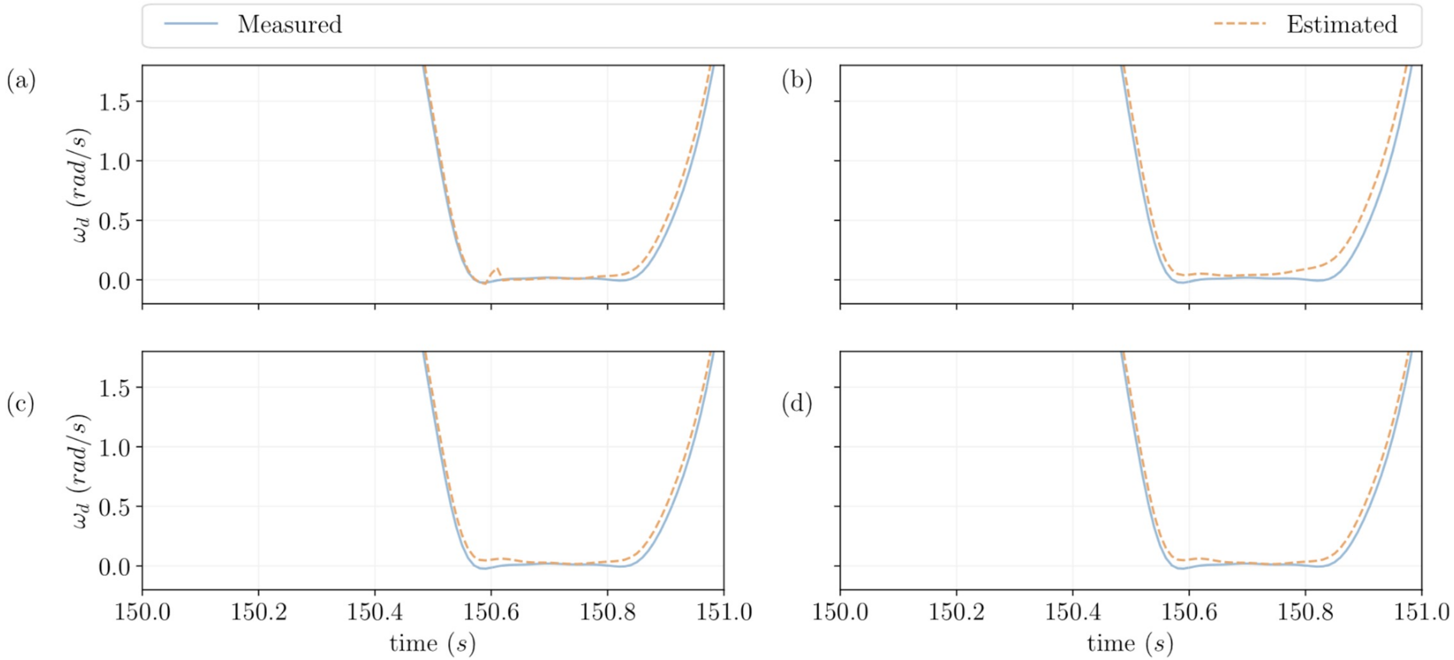

Fig. 7. Comparison of measured and predicted disc angular velocity using ensemble model based on (9), (8) and (14), one stick phase interval: (a) Coulomb; (b) Stribeck; (c) Dahl; (d) Stefanski et al.

Mechanical Systems and Signal Processing 200 (2023) 110405

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Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Check for updates

Hybrid gray and black-box nonlinear system identification of an elastomer joint flexible robotic manipulator

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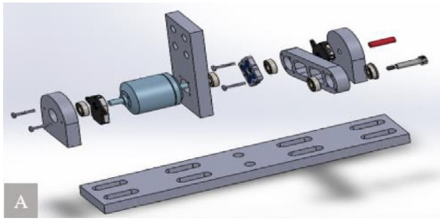
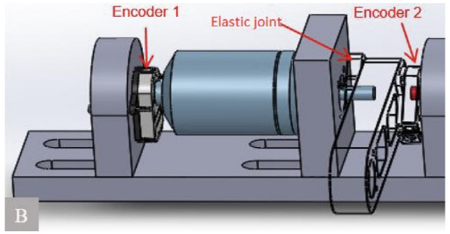
Goals:

- Build physics-based and black-box models
- Improve results by merging approaches

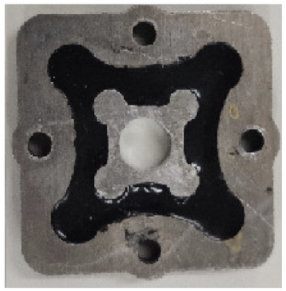
Flexible-joint manipulator



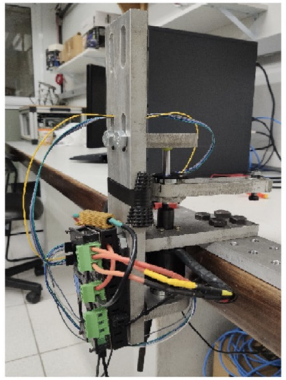
(a) Assembly exploded view [40]



(b) Assembly detail [40]



(c) Elastomer based elastic joint 55A



(d) Assembled eSEA

Fig. 1. eSEA assembly including CAD model, a detailed picture of the elastomer-based compliant element and the assembled system.

Physical models

$$\begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} \cdot \begin{bmatrix} \ddot{\delta} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} K_e & -K_e \\ -K_e & K_e \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \theta \end{bmatrix} + \begin{bmatrix} F_f\left(\dot{\delta}\right) \\ 0 \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$

$$F_{f_{viscous}} = f_v \dot{\delta}$$

$$F_{f_{Coulomb}} = f_v \dot{\delta} + \left[f_c + \left(f_s - f_c \right) e^{-\left(\dot{\delta}/\dot{\delta}_s\right)^2} \right] sign\left(\dot{\delta}\right)$$

$$F_{f_{Dahl}} = \sigma_0 z$$

$$\dot{z} = \dot{\delta} \left[1 - \frac{\sigma_0 z}{f_c} sign\left(\dot{\delta}\right) \right]$$

$$F_{f_{LuGre}} = \sigma_0 z + \sigma_1 \dot{z} + f_v \dot{\delta}$$

$$\dot{z} = \dot{\delta} \left[1 - \frac{\sigma_0 z}{f_c + \left(f_s - f_c \right) e^{-\left(\dot{\delta}/\dot{\delta}_s\right)^2}} sign\left(\dot{\delta}\right) \right]$$

Hybrid model: grey and black-box model

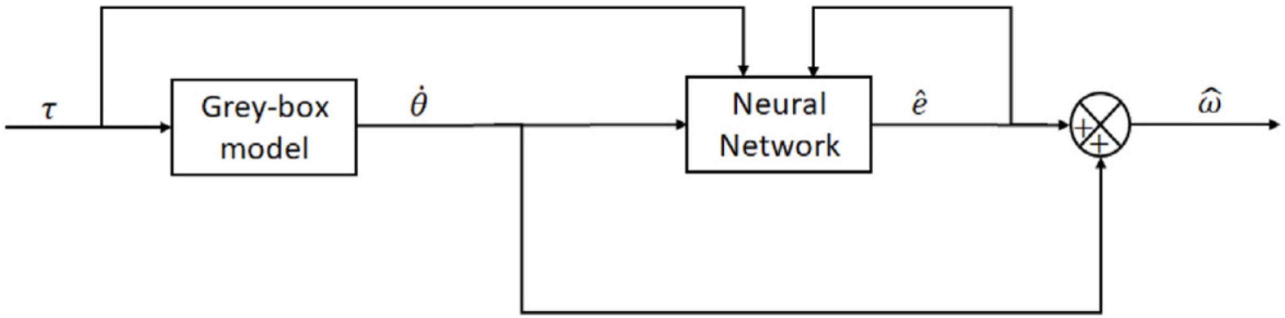


Fig. 6. Block diagram of the proposed hybrid model.

$$\hat{e}(t) = F[\hat{e}(t-1), \ldots, \hat{e}\left(t - n_{\hat{e}}\right), \tau(t-1), \ldots, \tau\left(t - n_{\tau}\right), \dot{\theta}(t-1), \ldots, \dot{\theta}\left(t - n_{\dot{\theta}}\right)]$$

$$\hat{\omega} = \dot{\theta} + \hat{e}$$

Results – hybrid enhaces prediction

Table 5
Model metrics comparison (Identification).

Model	MSE	R^2	MSE reduction
Linear	3.3253	0.8989	84.91%
Hybrid model (Linear)	0.5018	0.9847	
Coulomb with Stribeck	4.5041	0.8630	89.42%
Hybrid model (Coulomb)	0.4765	0.9855	
Dahl	4.1782	0.8729	47.30%
Hybrid model (Dahl)	2.2018	0.9330	
LuGre	1.7023	0.9482	60.95%
Hybrid model (LuGre)	0.6648	0.9798	

Heterogeneous data sources

Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 58-15 (2024) 514–519

Identification of the friction model of a single elastic robot actuator from video

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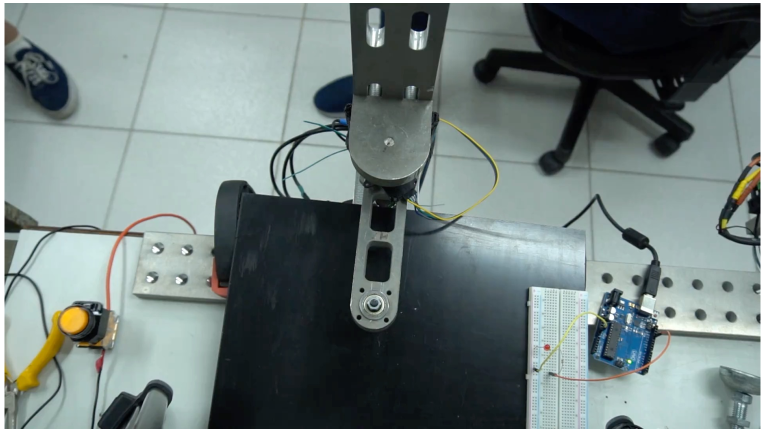
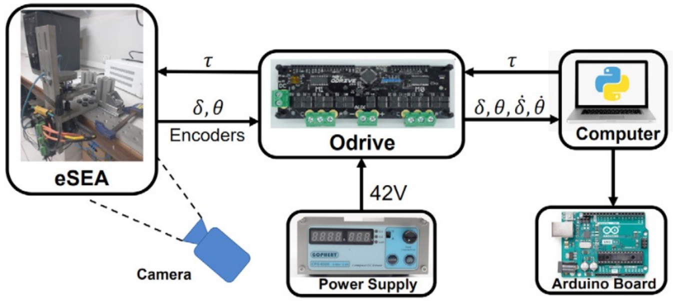
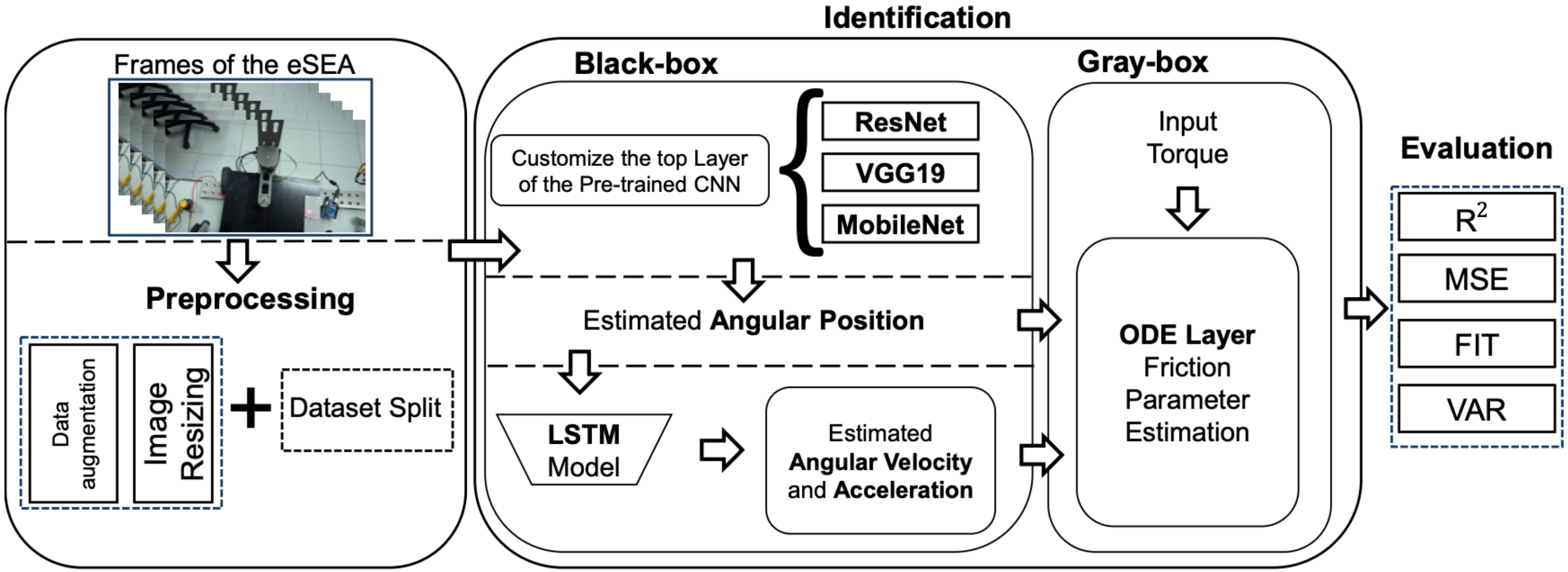
[**] *Instituto Militar de Engenharia, RJ, Brasil, (e-mail: braz.daniel@ime.eb.br).*
[***] *Department of Statistics and Data Science, Yale University, New Haven, CT 06511, USA, (e-mail: lu.lu@yale.edu).*
[****] *Pontifícia Universidade Católica do Paraná, PR, Brasil, (e-mail: helonayala@gmail.com).*

2024 IFAC SYSID – BOSTON, USA





Hybrid models and Heterogeneous data sources



Takeaways

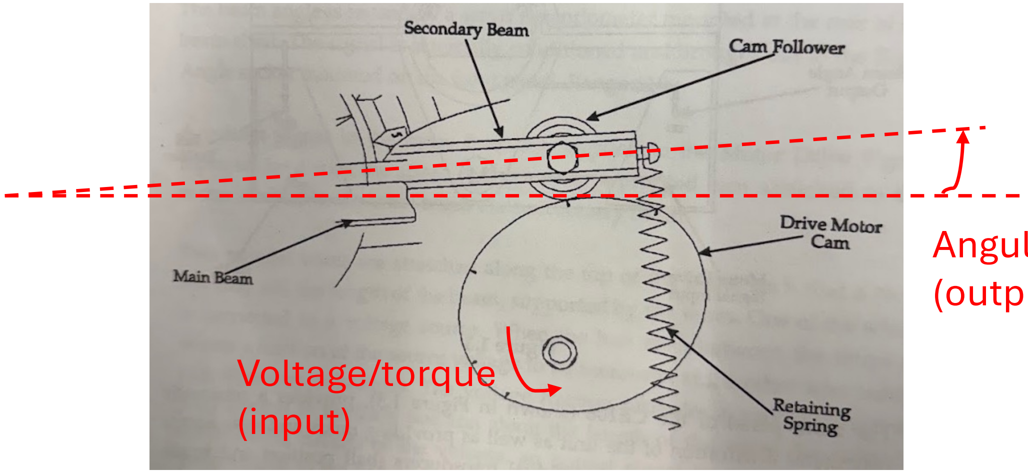
- Hybrid architecture varies greatly throughout examples
- It is necessary to establish which approaches are better
  - Simulation performance
  - Model-based control
- Multimodal modeling seems to be difficult

Future/ongoing work

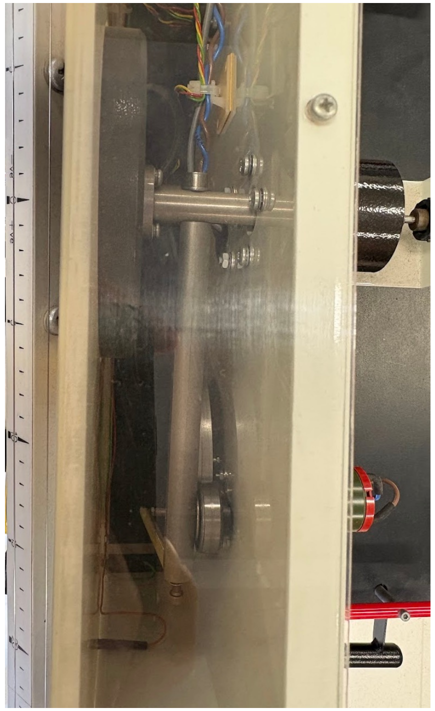
Case study

- A beam that rotates commanded by a DC motor on a cam follower

Schematic



Cam follower detail

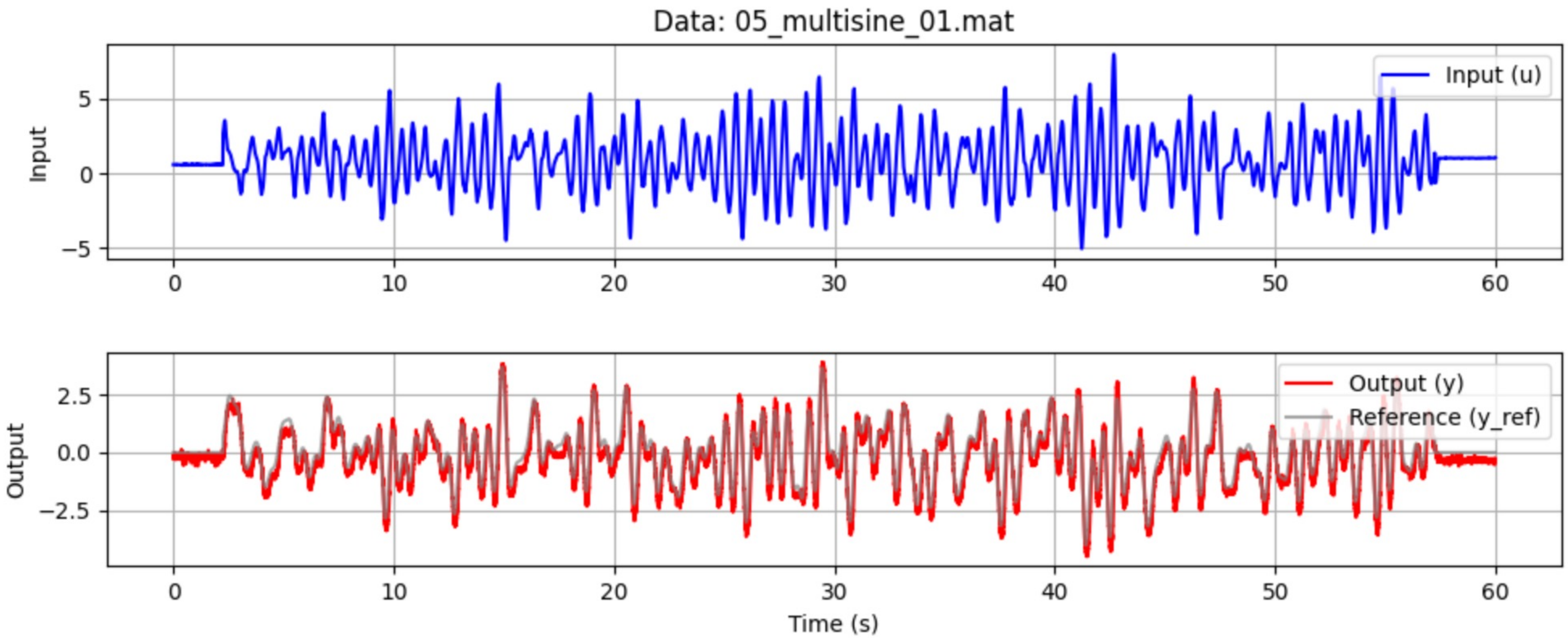


Motivation

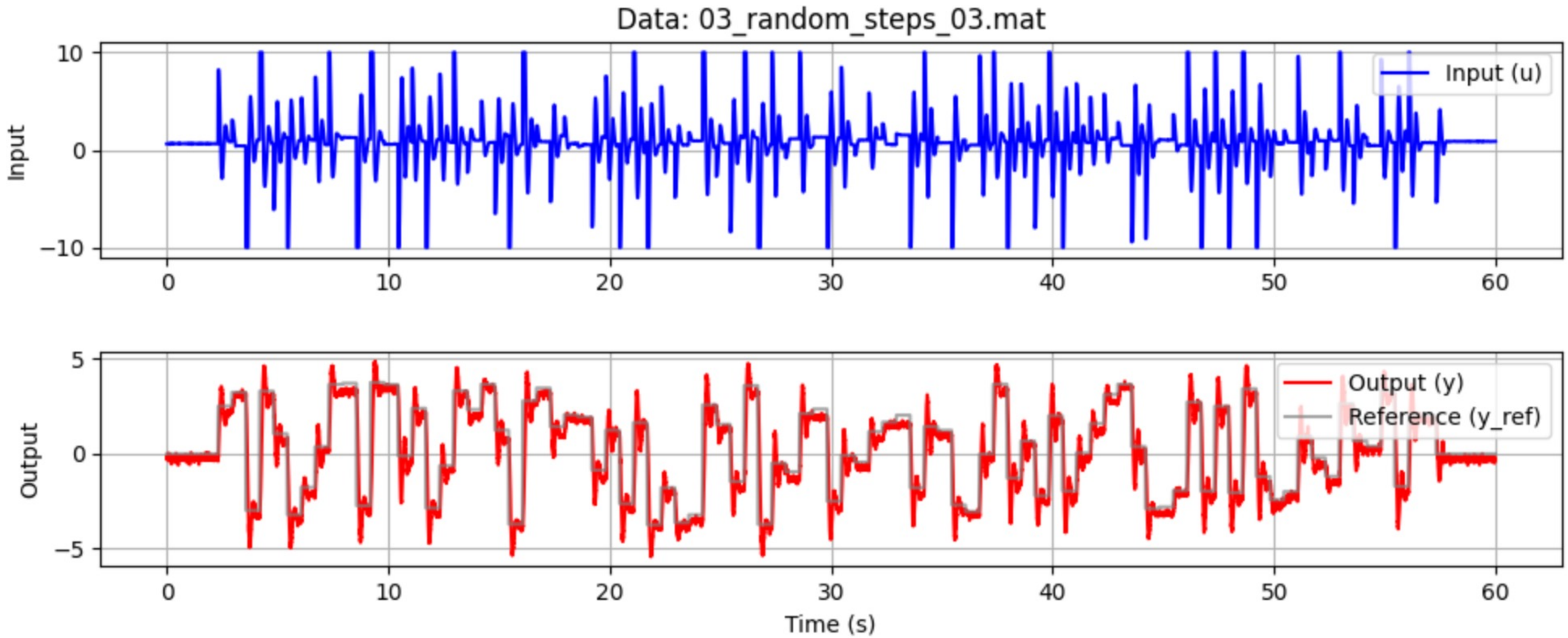
- Nonlinear phenomena
  - Came geometry, friction and dead-band
- Establish a benchmark for multimodal SYSID
  - Comparison!
- Classic control example, easy to grasp
- Enables testing concepts for scaling

https://github.com/helonayala/bab_datasets

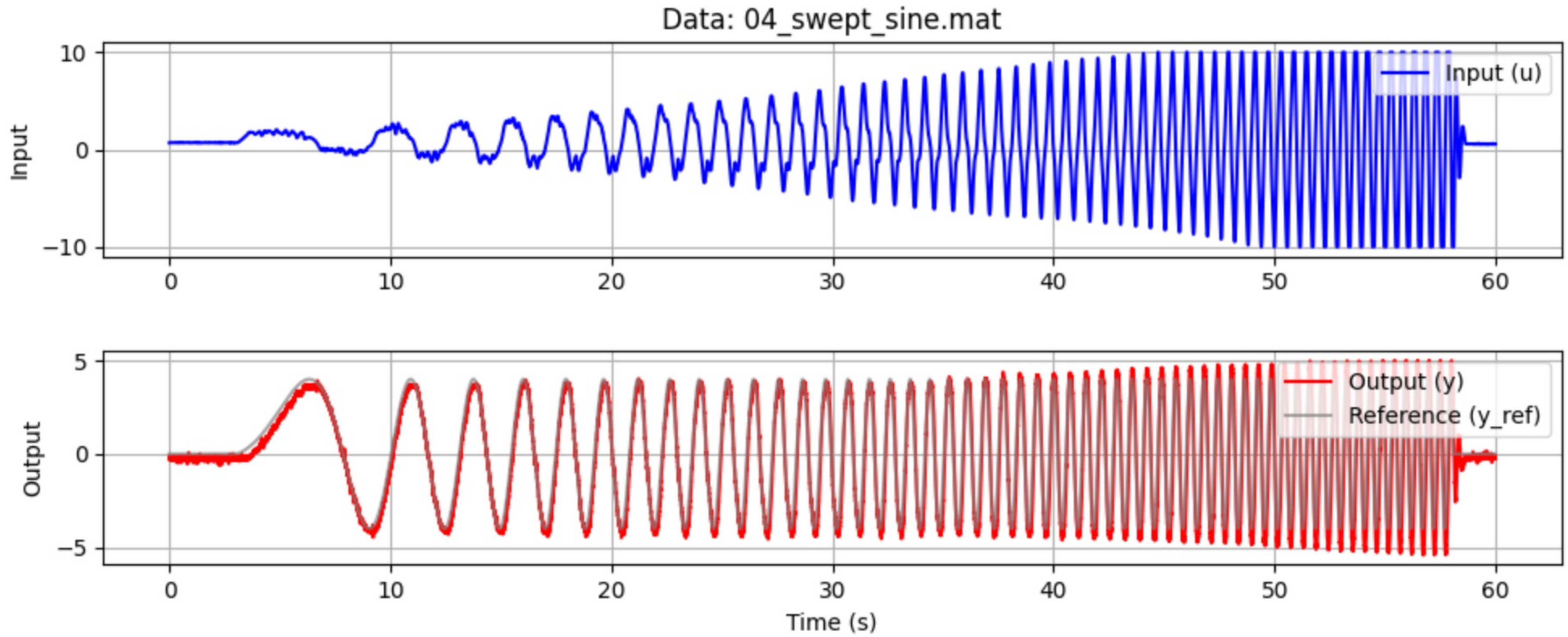
Dataset 1-2: multisine (2x)



Dataset 3-6: random-steps (4x)

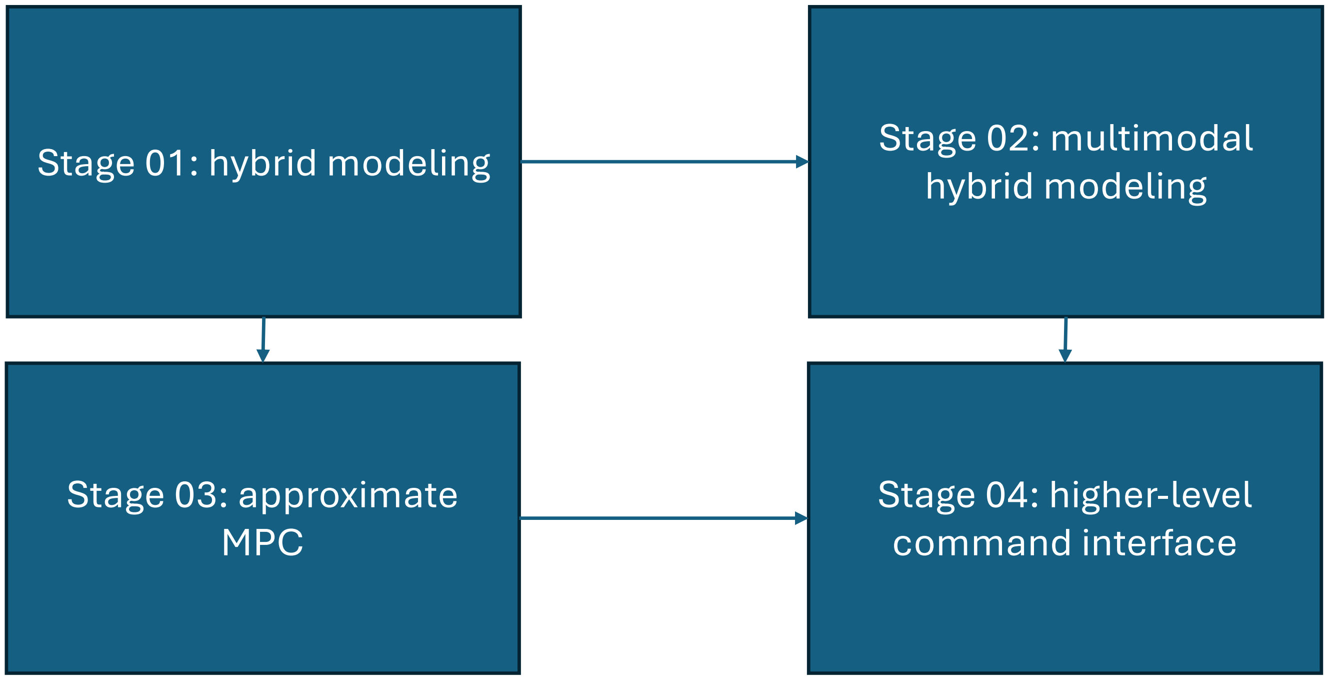


Dataset 7: swept-sine



ERC/CNPq Project Goals

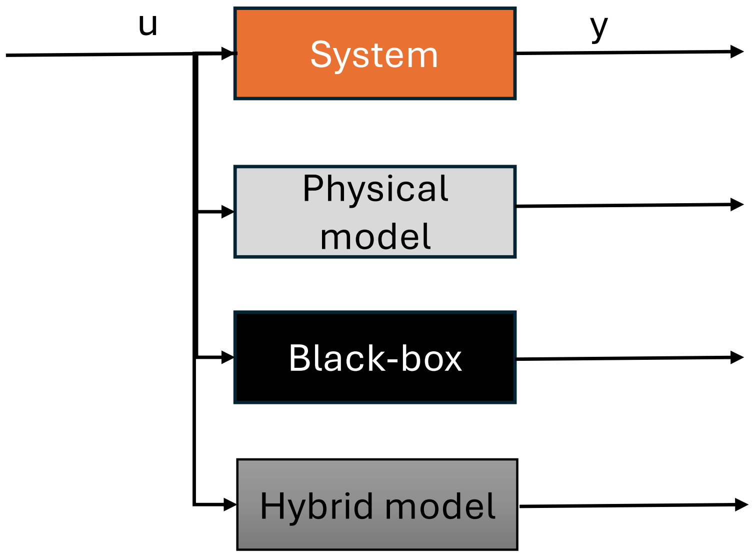
Hybrid system identification for approximate MPC



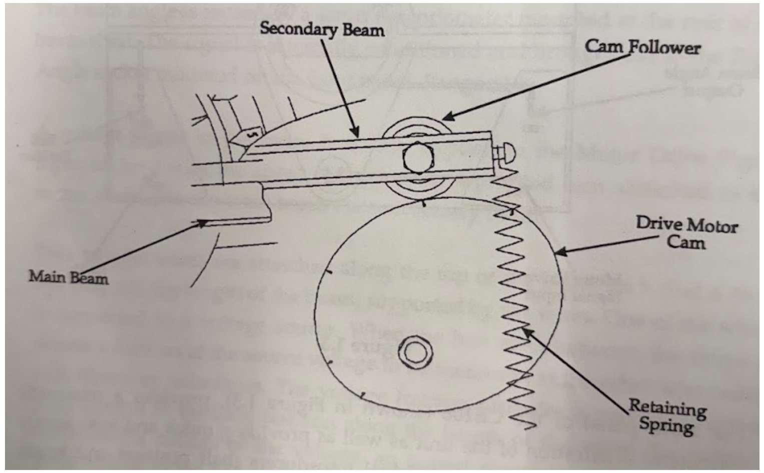
Stage 01: hybrid modeling

- Goal:
  - improve hybrid modeling wrt. prior work

- Impact:
  - establish novel hybrid architectures
  - improve model-based control



Stage 01: models tested



- Linear $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{J_0}\left(\tau_0 u(t) - R_0 x_2 - K_0(x_1 + \delta_0)\right) \end{cases}$$
- Nonlinear $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{J_0}\left(\tau_0 u(t) - K_0(x_1 + \delta_0) - \left[F_c + (F_s - F_c)e^{-(|x_2|/v_s)^2} + \sigma x_2\right]\right) \end{cases}$$
- Black-box $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \mathrm{NN}(x_1, x_2, u(t); \theta_{net}) \end{cases}$$
- Hybrid $$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \underbrace{\frac{1}{J_0}\left(\tau_0 u(t) - R_0 x_2 - K_0(x_1 + \delta_0)\right)}_{\text{Physical}} + \underbrace{\mathrm{NN}(x_1, x_2, u(t); \theta_{net})}_{\text{Residual}} \end{cases}$$

Stage 01: models tested

- Mini-batches are built from K-step rollouts
  - Full-simulation vs. 1-step ahead

- Loss function
  - error is calculated only for **position (measured data)**
  - **velocity (estimated)** is used only for IC

Stage 01: preliminary results

Model/R2	**ms_05**	**ms_06**	**rs_01**	**rs_02**	**rs_03**	**rs_04**	**swept**
Linear	0.8511	0.8404	0.7439	0.6846	0.5800	0.3755	0.8637
Stribeck	0.8578	0.8480	0.7540	**0.7161**	0.6381	0.4907	0.8801
Black-box	0.8794	0.6762	**0.7686**	0.7068	**0.6915**	**0.5986**	0.8055
Hybrid-Joint	0.9467	**0.9643**	0.5601	0.5110	0.4078	0.4406	**0.8984**
Hybrid-Frozen	**0.9821**	0.9316	0.5677	0.4854	0.3235	0.4899	0.8914

Protocol 1

- Train
  - Multisine
- Test
  - Random steps & Swept-sine

- Broadband signals: hybrid is better
- Random-steps present poor results
  - Signals used for training have different properties than those used for testing

Stage 01: preliminary results

Model/R2	ms_05	ms_06	rs_01	rs_02	rs_03	rs_04	swept
Linear	0.4025	0.6423	0.7822	0.7475	0.5875	0.5626	0.8679
Stribeck	0.5382	0.4376	0.4586	0.4344	0.2880	0.2520	0.6116
Black-box	-3.9730	-1.9576	0.5234	0.5843	0.4914	0.6467	0.8299
Hybrid-Joint	**0.9122**	**0.8645**	**0.9318**	**0.7863**	**0.6329**	**0.8895**	**0.9532**
Hybrid-Frozen	0.7983	0.6804	0.7266	0.6004	0.5082	0.6440	0.5814

Protocol 2

- Train
  - 50% of multisine and random-steps
- Test
  - 50% of multisine and random-steps
  - Sine-sweep

- Hybrid is better
- Joint-estimated parameters perform better

Stage 01: take-aways / future work

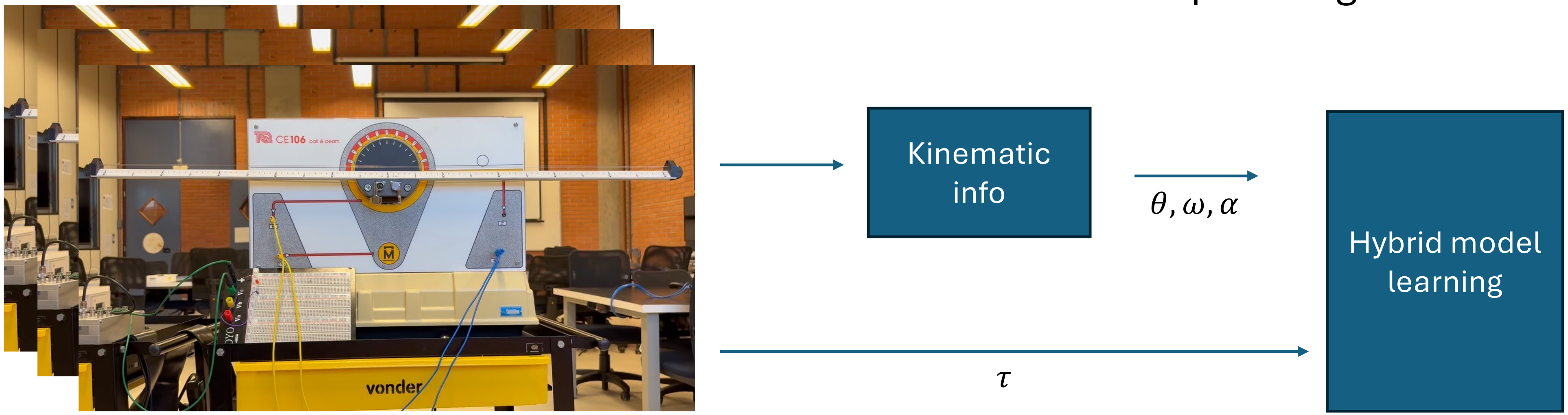
- Run hyperparameter extensive search

- Reiterate physical model
  - Deadband / backlash; cam follower geometry

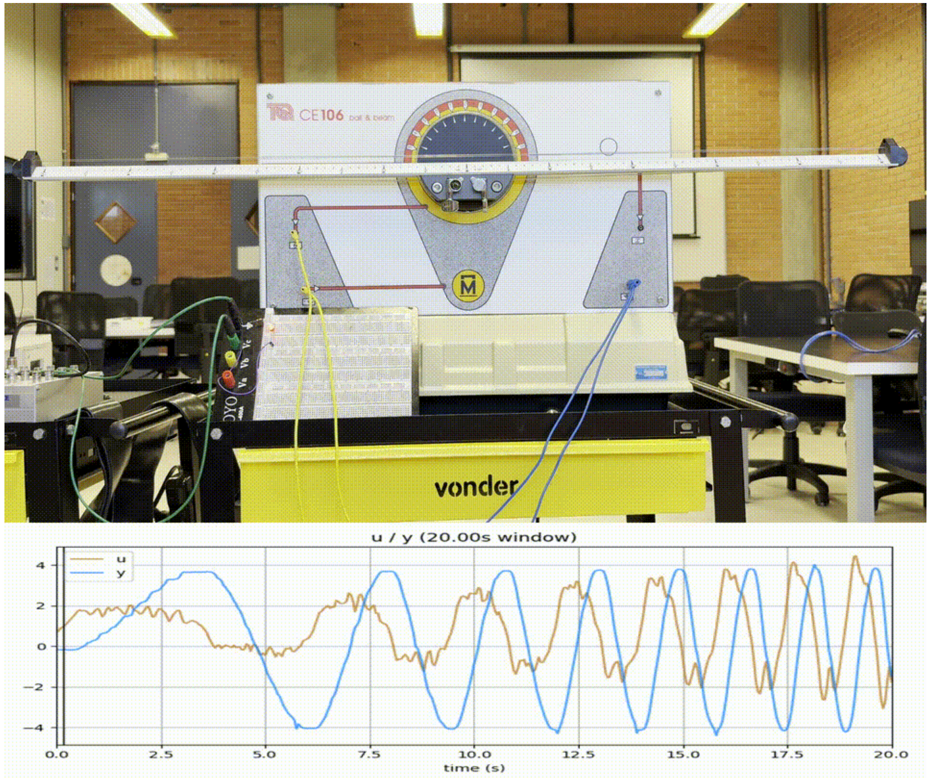
- Information contained in the datasets
  - Is it possible to generate a good model looking solely at the data?

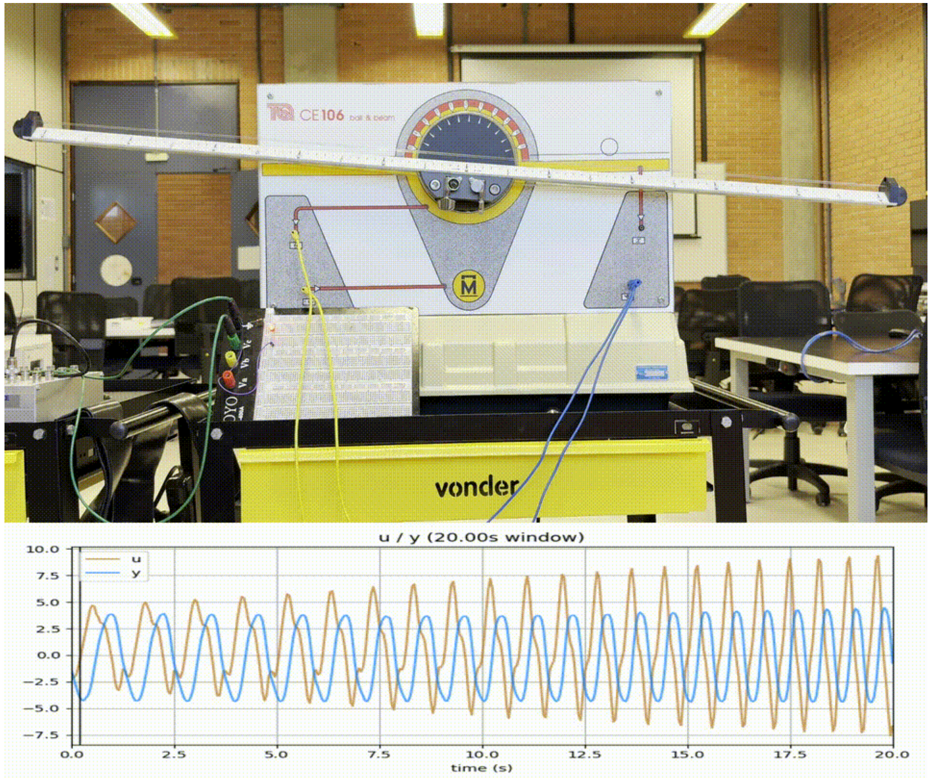
Stage 02: multimodal hybrid modeling

- Goal:
  - Build hybrid models with video
- Impact:
  - allow predicting using video
  - simulations output images



Stage 02: video sync





Stage 03: approximate MPC

- Goal:
  - Run MPC online using hybrid models
- Impact:
  - If hybrid is more accurate, MPC improves

$$J_N(u_{k,k+N-1}) = \sum_{i=k}^{k+N-1} \|x_i - q_i\|_Q^2 + \sum_{i=k+1}^{k+N-1} \|\Delta u_i\|_S^2$$

$$u^*_{k,k+N-1} = \arg\min J_N(u_{k,k+N-1}),$$

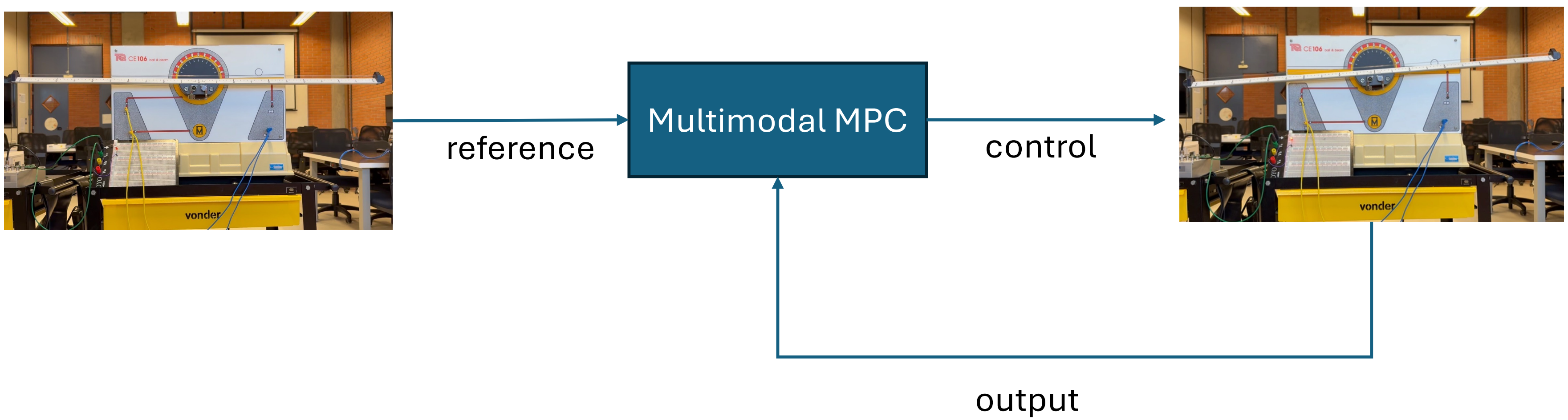
$$\hat{u}_k = NN_\theta(x_k, q_{k,k+N-1})$$

Ortega and Camacho, *Mobile robot navigation in a partially structured static environment, using neural predictive control*, **Control Engineering Practice**, 4(12), pp. 1669-1679, 1996.

Hose, Weisgerber and Trimpe, *The Mini Wheelbot: A Testbed for Learning-based Balancing, Flips, and Articulated Driving*, **ICRA**, 2025.

Stage 04: higher-lever command interface

- Goal:
  - Implement MPC with image-based prediction model
- Impact:
  - Allow human-like interfacing with low-level hardware



Hybrid Modeling and System Identification: Past and Future Directions

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FAU MoD Lecture

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Friedrich-Alexander-Universität
DYNAMICS, CONTROL, MACHINE LEARNING AND NUMERICS